Fast computation of general direct gravitation problems

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In this work, we derive a fast algorithm for the computation of the gravitation effects of a general volumic mass distribution. The algorithm consists in first computing a tetrahedral mesh modeling the sources. Then, the gravitation effects, namely the gravity potential and its first and second derivatives, are computed using a quadrature formula when the observation points are far away from the sources, and the Okabe analytical formulae when the observation points are close to the sources.

The novelty of this work consists in the use of a fast algorithm that accelerates the long-range interactions: the Fast Multipole Method (FMM). The algorithmic complexity is reduced from quadratic to linear, with respect to the data (sources and observation points), making available large test cases that are intractable with classical methods. Our implementation makes use of Scalfmm [1], a powerful parallel implementation of the FMM developed by INRIA, and is ready to use of large clusters. The approximations made in this algorithm are controllable.

When the observation points are far enough from the sources, all the interactions can be computed using a one-point quadrature formula, leading to fast execution times. This was applied to the computation of the global topographical effect on gravity at the GOCE satellite altitude. After building – in linear complexity – a mesh of the difference between the real Earth and the WGS84 reference ellipsoid composed of 100 million tetrahedrons, any component of the gravitation effects over 2 million observation points are computed in 6min30s on a 16 processors cluster.

References

[1] O. Coulaud, B. Bramas, and C. Piacibello. Scalfmm, C++ Fast Multipole Method Library for HPC. *http://scalfmm-public.gforge.inria.fr/doc/*.