A new homogenization method for the atmospheric GNSS data

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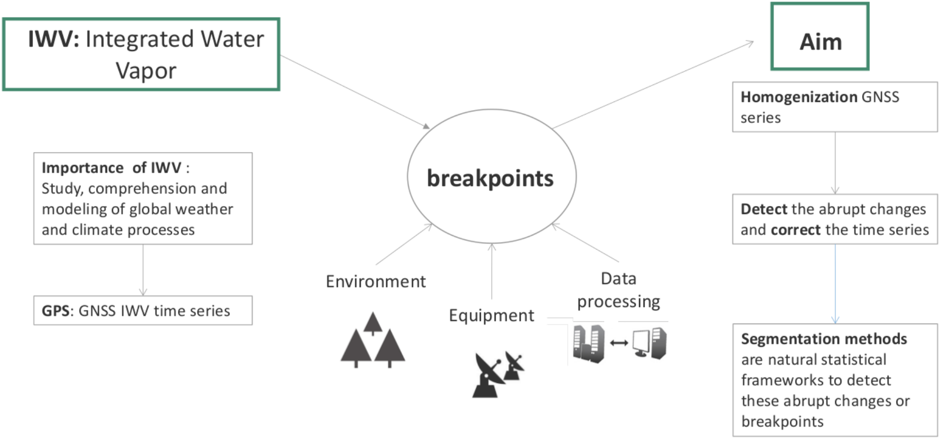
Outline

GNSS Time Series and Objective



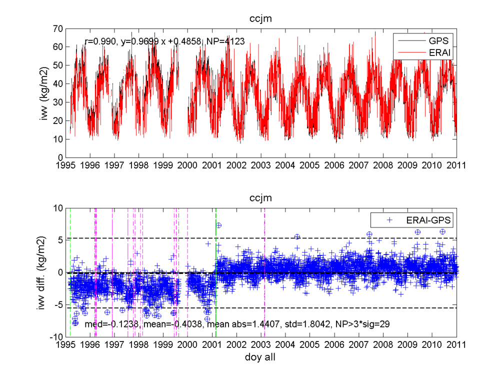
Study of IWV

1. Detection problem: finding break-points in the IWV time series

2. Adjustment problem: correcting the time series

3. Climate analysis on the clean IWV series

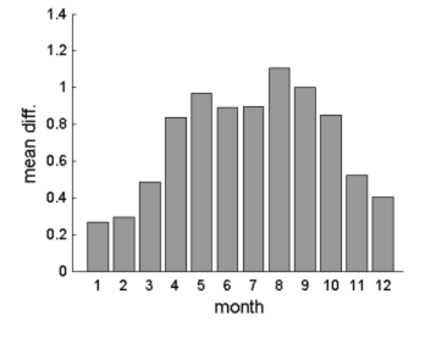
Example of Such Data

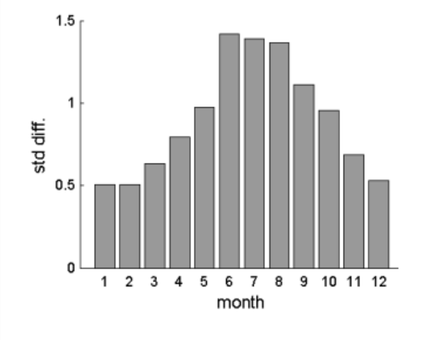


- Natural seasonal tendency
- ERAI : meteorological reanalysis
- $\Delta IWV = IWV_{ERAI} - IWV_{GPS}$

Features of the Data

Monthly mean and variance for the station CCJM

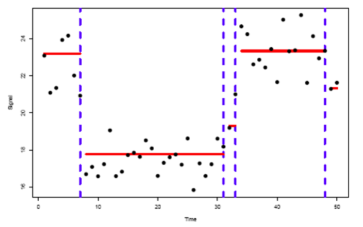




Features of the data.

1. a periodic signal remains
2. a non-stationary of the variability is observed

Breakpoint detection in the mean model with homogeneous variance



Example of segmentation with homoscedastic model

Model. The signal $Y = (Y_1 \ldots, Y_n)$ is such that $Y_t = \mu_k + E_t$,

$$\forall t \in I_k = [t_{k-1} + 1; t_k] \text{ where}$$

$$\{E_t\}_t \quad are \quad i.i.d. \sim \mathcal{N}(0, \sigma^2).$$

Parameters. The $K - 1$ breakpoints $T = \{t_1, \ldots, t_{K-1}\}$, the means and the variance $\theta = \{\mu_1, \ldots, \mu_k, \sigma^2\}$ and the number of segments K.

Classical Inference Strategy: two steps

1. K being fixed, estimation of T and θ by maximum likelihood
   - ⋆ There are $\binom{n-1}{K-1}$ possible segmentations of n points into K segments
   - ⋆ Dynamic Programming (DP) algorithm gives the exact solution in a reasonable complexity time $O(Kn^2)$
   - ⋆ DP can be applied if and only if the quantity to be optimized is additive with respect to the segments

2. Choice of K. Model selection issue: Penalized log-Likelihood.

A New Segmentation Model

$$Y_t = \mu_k + f_t + E_t,$$

$$\forall t \in I_k = [t_{k-1}+1; t_k] \cap I_{\text{month}} = \{t; \text{date}(t) \in \text{month}\}$$

where

- ⋆ f_t will be approximated using a Fourier decomposition of order $4 \rightarrow$ to take into account for the periodic signal

$$f_t = \sum_{i=1}^{4} \left(a_i \cos(i2\pi t/L) + b_i \sin(i2\pi t/L)\right) \quad \text{with } L \text{ the length of a year}$$

- ⋆ $\{E_t\}_t \quad i.i.d. \sim \mathcal{N}(0, \sigma^2_{month}) \rightarrow$ to take into account for the monthly variability

Proposed Inference Strategy $\rightarrow$ to allow the use DP

1. Estimation of the variances σ^2_{month} using a robust approach (robust to the breakpoints) $\rightarrow \hat{\sigma}^2_{month}$

2. Segment the signal with 'known variances'

   ⋆ K being fixed, an iterative procedure, at iteration $[h+1]$:

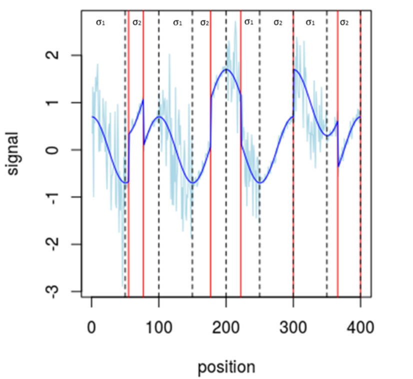
     – Estimation of f on $\tilde{y}_t = y_t - {\mu_k}^{[h]}$ using a weighted least square regression with weights $1/\hat{\sigma}^2_{month}$

     – Estimation of t_k, μ_k on $\tilde{y}_t = y_t - f_t^{[h+1]}$ using DP

   ⋆ Choice of K by adapting model selection criteria (BM , Lav and mBIC)

Simulation Design

- ⋆ $n = 400$
- ⋆ $\sigma_1 = (0.1, 0.5, 0.9)$
- ⋆ σ_2 from 0.1 to 1.5 (by step 0.2)
- ⋆ $f_t = 0.7\cos(\frac{2\pi t}{100})$
- ⋆ $K = 7$
- ⋆ $T = [55, 77, 177, 222, 300, 366]$
- ⋆ $\mu = [0, 1, 0, 1, 0, 1, 0]$



Example of simulated data

Quality Criteria

- ⋆ The difference between the predicted number of segments and the true number one: $\hat{K} - K_{True}$

- ⋆ The root mean squared error between the function f and its estimate :

$$\text{RMSE}(f) = \left[\frac{1}{n} \sum_{t=1}^{n} \left\{ f_t - \hat{f}_t \right\}^2 \right]^{1/2}$$

- ⋆ The root mean squared error between the mean μ and its estimate : $\text{RMSE}(\mu)$

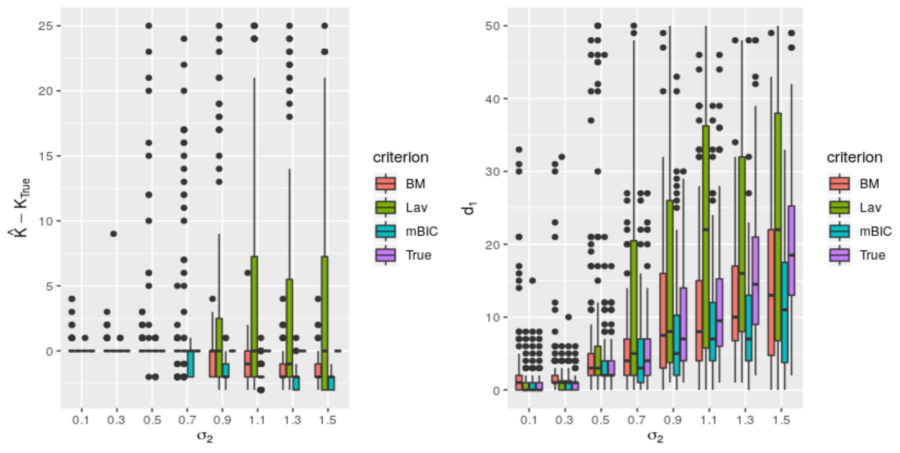
- ⋆ Two components of Hausdorff distance: $d_1(\hat{T}, T_{True})$ and $d_2(\hat{T}, T_{True})$, where

$$d_1(a, b) = \max_b \min_a |a - b|, d_2(a, b) = d_1(b, a)$$

$\rightarrow$ A perfect segmentation results in $d_1 = d_2 = 0$

Simulation Results

Results for $\sigma_1 = 0.5$ and $\sigma_2 = (0.1, 0.3, \ldots, 1.5)$



Simulation Results

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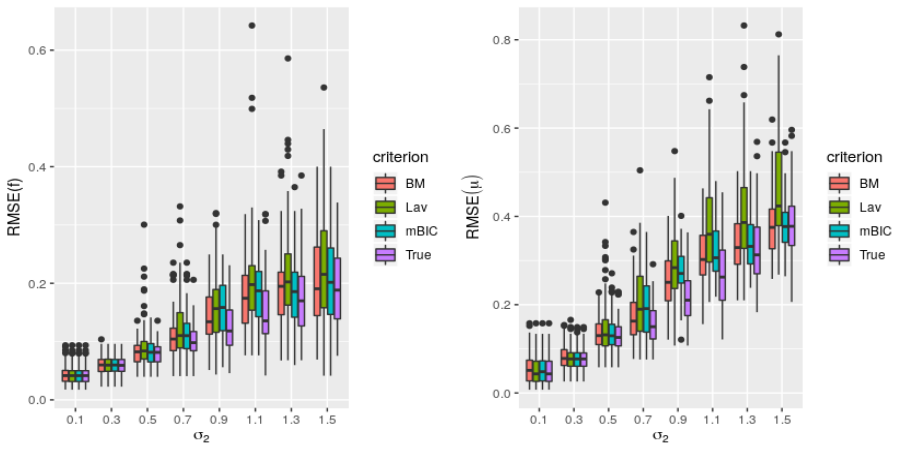
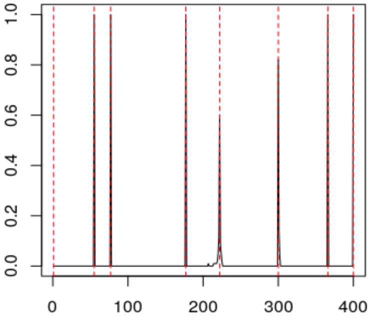


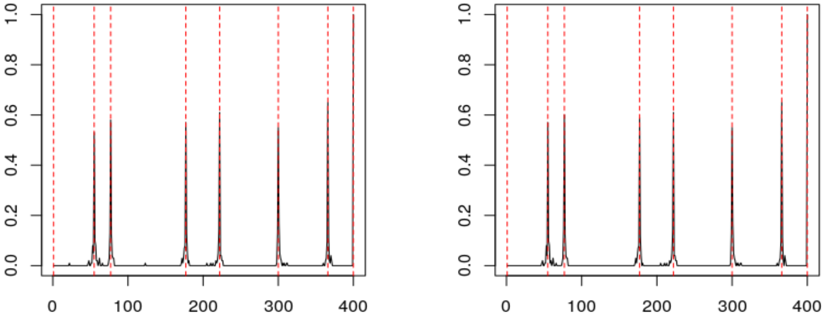
Simulation Results

Breakpoints positioning. Frequencies of all possible breakpoints estimator for $\sigma_1 = 0.5$ and $\sigma_2 = 0.1$ for K_{mBIC} and K_{True}



Simulation Results

Breakpoints positioning. Frequencies of all possible breakpoints estimator for $\sigma_1 = 0.5$ and $\sigma_2 = 0.5$ for K_{mBIC} and K_{True}



Simulation Results

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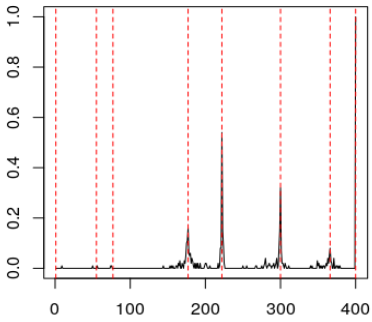
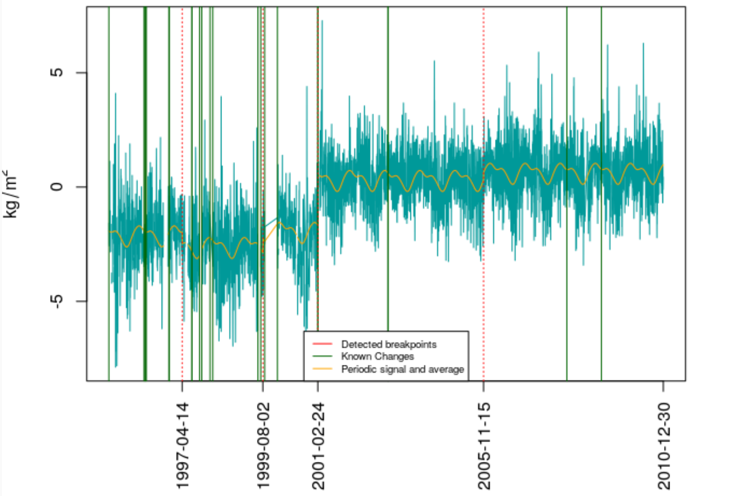


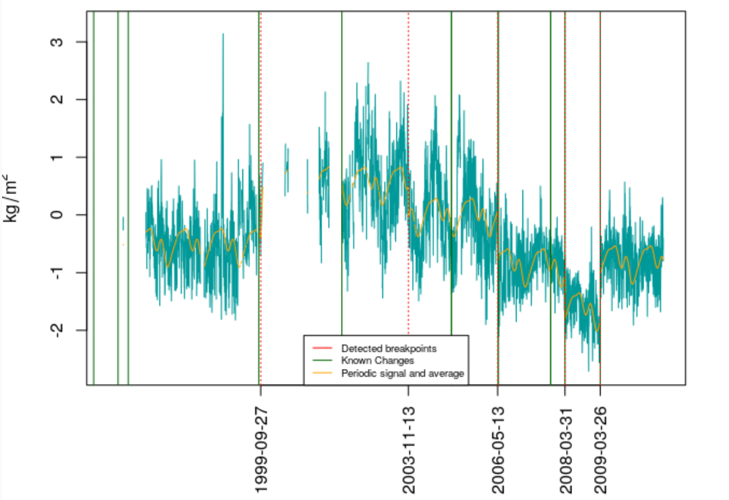
Application of Real Data

Station CCJM with selection criterion BM



Application of Real Data

Station MCM4 with selection criterion BM



Conclusion and Further Developments

Conclusion

1. We proposed a segmentation method for the analysis of GNSS time series: a breakpoint detection in the mean model in which
   - the variance is supposed to be month-dependent
   - a periodic signal is taken into account
2. We developed an R Package, **GNSSseg** available on GitHub: https://github.com/arq16/GNSSseg.git
3. Two articles on the way (Bock in press and Quarello in preparation)

Further developments

- Improvement of the estimation of f: non-parametric approach
- Take into account for a time-dependence that may exist on time series

References

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Merci!

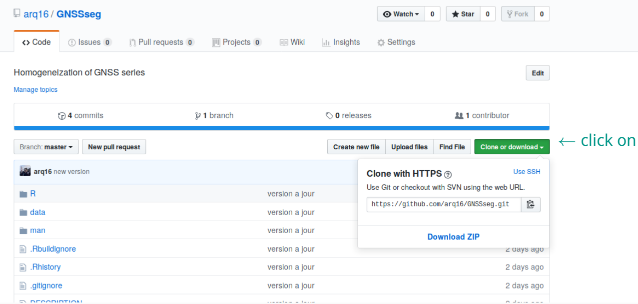
outline

GNSSseg R Package

How to install an R Package from GitHub?

```
library(devtools)
devtools::install_github("https://github.com/arq16/GNSSseg.git")
```

- https://github.com/arq16/GNSSseg.git



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