Gravity field modeling in space and time

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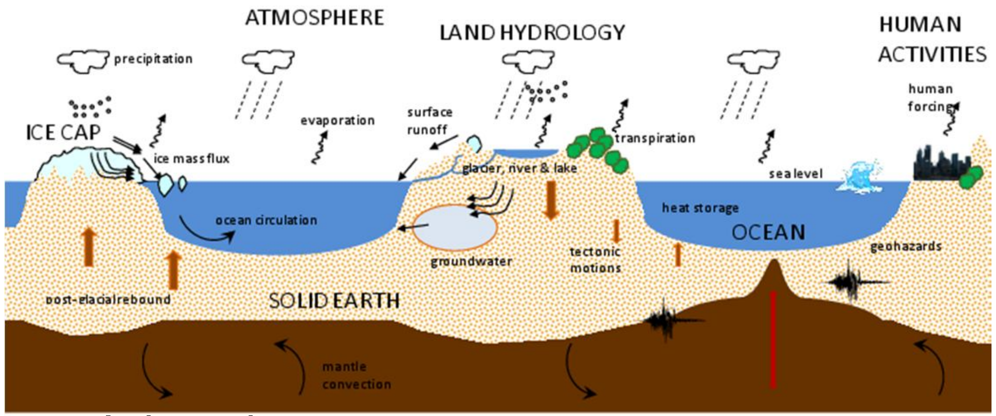
Logos - affiliations

Structure of the presentation

- Context and objective
  - Earth's masses transferts & satellite gravity
  - Objective
- 4D gravity field modeling
  - Space-time basis functions
  - Model estimation
- Results
  - Synthetic tests description
  - 4D gravity and water height model

Context

Masses move within the Earth's system, reflecting dynamic processes: earthquakes, post-glacial rebound, water displacement in the fluid envelopes, ...

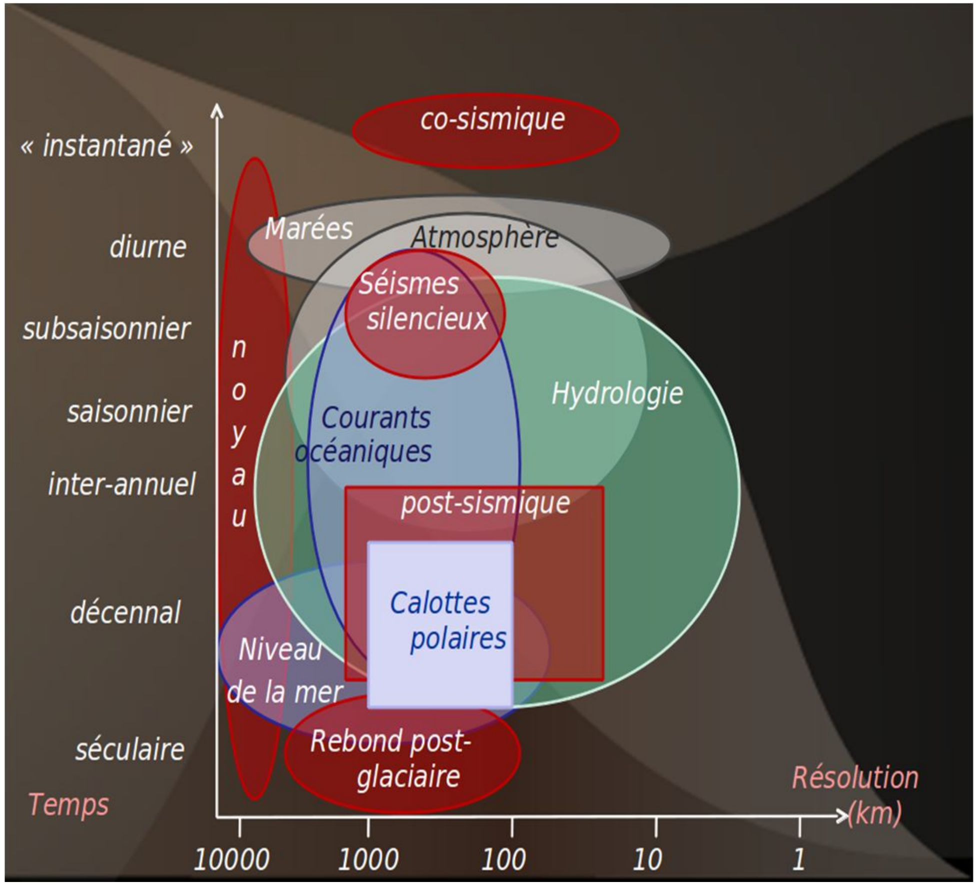


Panet et al. (2012)

These masses displacements create temporal variations of the gravity field

The Earth's system and the global water cycle

Multi-scale structure of Earth's gravity



These processes have different spatial and temporal scales

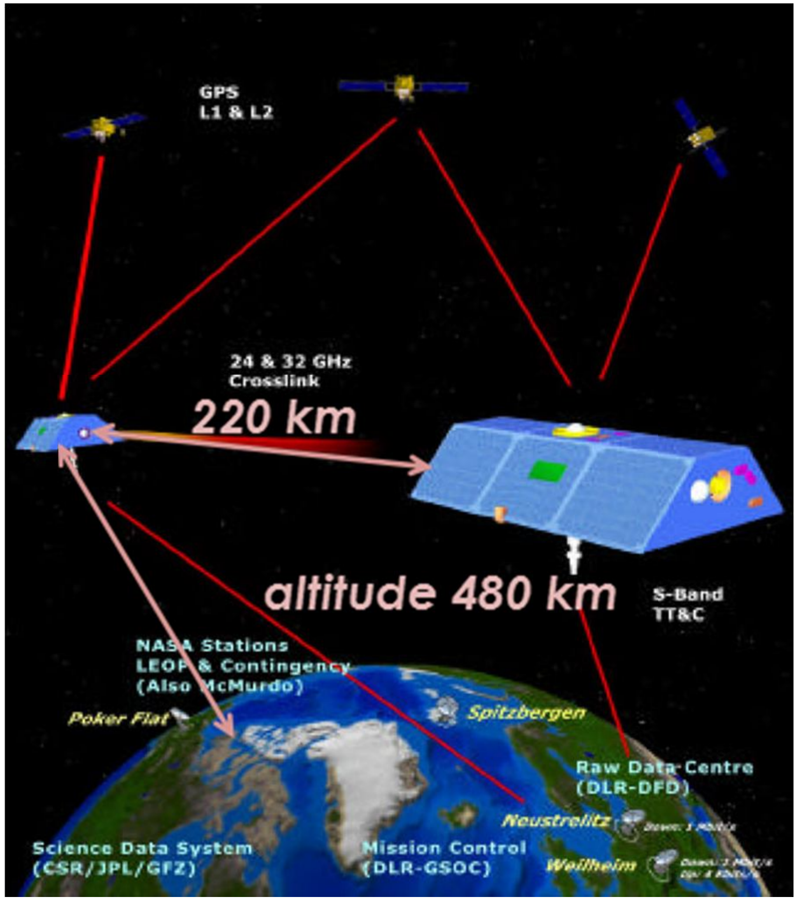
→

Gravity field variations in different spatial and temporal scales.

Satellite gravity missions

- Since 2002, satellite missions provide an unprecedented view of the gravity field spatial and temporal variations.
- Gravity models are built from these satellite data to study the Earth’s dynamic processes (Tapley et al., 2004).
- We focus on the time-varying geoid models from GRACE.

The GRACE mission



@NASA

A NASA/DLR mission, launched in 2002.

- The distance between the two satellites varies because the intensity of the gravity field varies from the position of the first satellite to the position of the second one.

- These distance variations are measured with a micrometric precision.

Objective

- In this work, we develop a fully 4D modeling of the gravity field in space and time, with application to GRACE.

- Objectives:
  - Adapt the temporal resolution to the spatial resolution: increase temporal resolution at large spatial scales / higher spatial resolution for long temporal scales
  - Describe local gravity variations, spatially and temporally

    ex: earthquakes, floods

→ **4D multi-scale models using localized basis functions**

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Space-time gravity models

We note $V(x,t)$ the gravitational potential at point x and time t:

$$V(x,t) = \sum_{k=1}^{M} d_k(t) g_k(x)$$

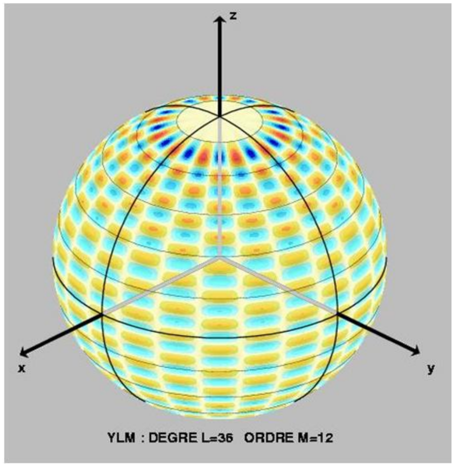
$g_k(x)$ are the spatial basis
$d_k(t)$ are the time dependent coefficient, developed as:

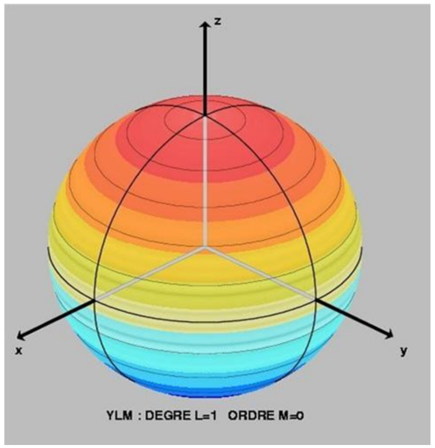
$$d_k(t) = \sum_{j=1}^{J_k} d_{k,j} h_j\,(t)$$

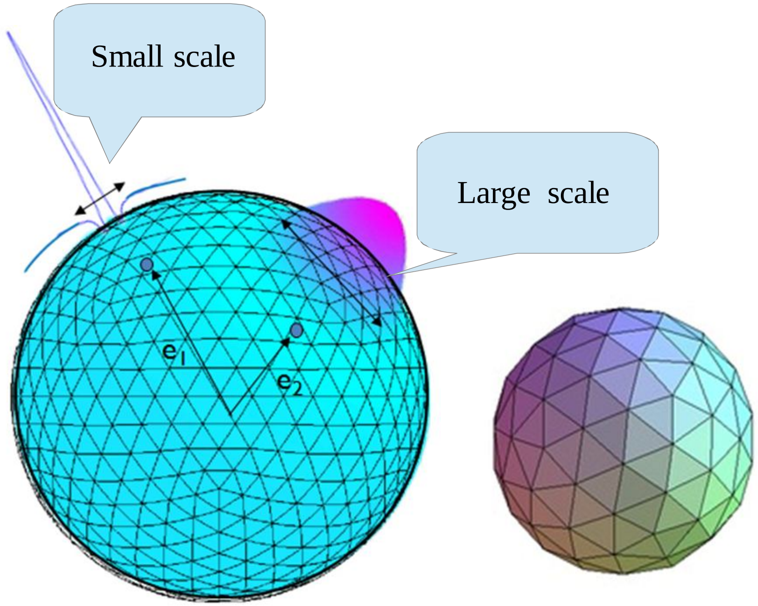
$h_j(t)$ are the temporal basis function.

The spatial basis

- Widely used approach: the spherical harmonics (global).
- Proposed model: localized wavelets functions.







Poisson waveletes

@Holschneider et al. 03

The temporal basis

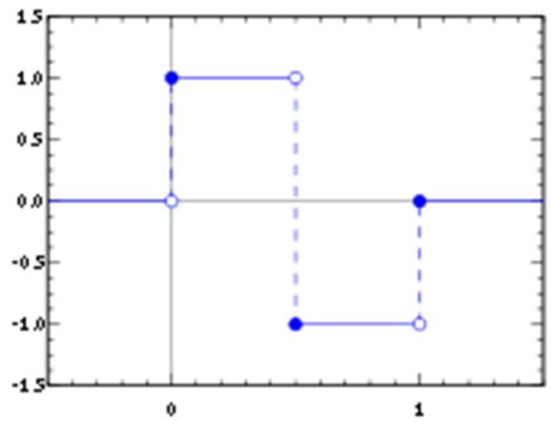
- Widely used approach:

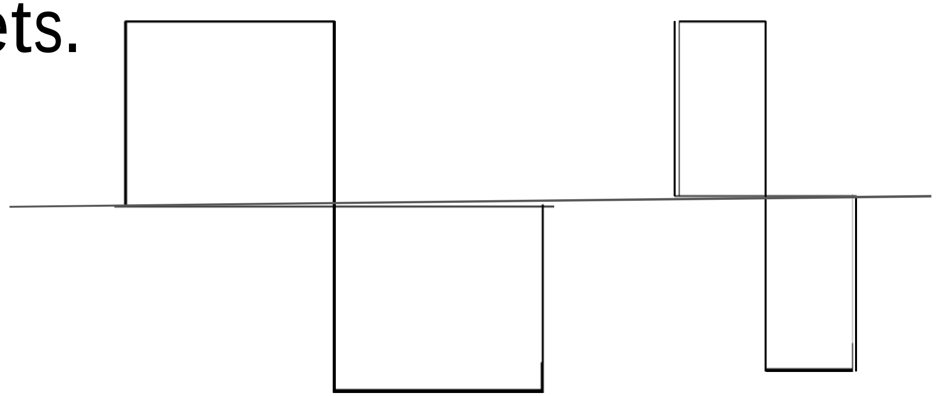
  1 month / 10 days averaging.

- Proposed model: temporal Haar wavelets.

  - localized ;
  - orthogonal (*simplifies the inverse problem*)

Mother function





large scale

small scale

Model estimation

- Inter-satellites geopotential differences along the satellites orbits DDP(x,y,t) are built using the mechanical energy conservation (Ramillien et al., 2011).
- Thus we have the following observation equations:

DDP(x,y,t) = V(x,t) – V(y,t)

GRACE B GRACE A

$$= \sum_{k=1}^{M} \sum_{j=1}^{J_k} d_{k,j} h_j\,(t)\,(g_k(x) - g_k(y))$$

Matrix notations: $DDP(X, Y, T) = A(X, Y, T).\,d$

Model estimation

- Regularized least-squares inversion:
  Minimize the misfit between modeled and observed DDP while requiring a smooth gravity potential:

$$Min_d(\| Ad - DDP \|_W{}^2 + \| d \|_K{}^2)$$

Data misfit

Smoothness constraint

- This leads to the normal system:

$$d = (A^t W^{-1} A + K^{-1})^{-1} A^t W^{-1} DDP$$

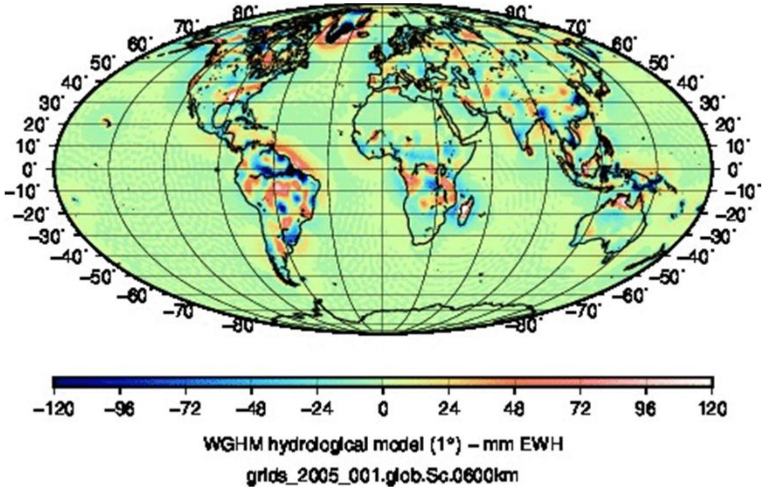
Block diagonal
1 block =
1 temporal
scale/position

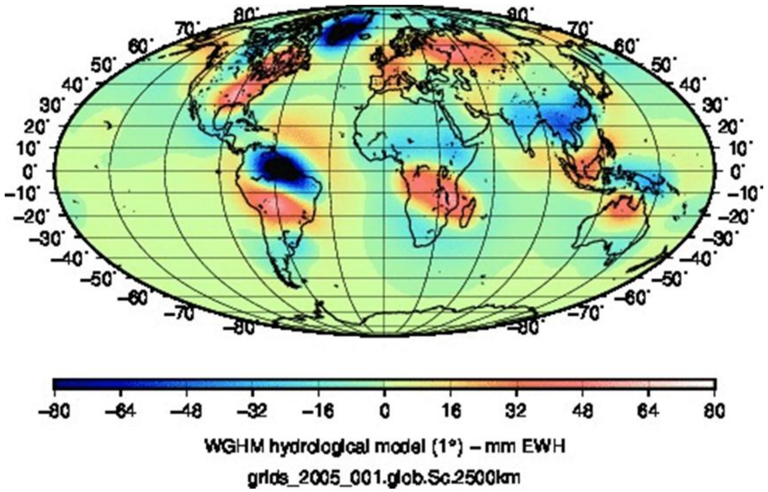
Noise prior covariance
matrix: c. Id

Wavelet coefficients
prior covariances

The regularization

- K: assumption of a diagonal matrix (we neglect the wavelets covariances)
- Prior 4D wavelets coefficients variances: obtained from the prior amplitude of the geopotential variations at each spatial and temporal scale.
- This prior knowledge is constructed from a 4D space-time analysis of the WGHM water height model.





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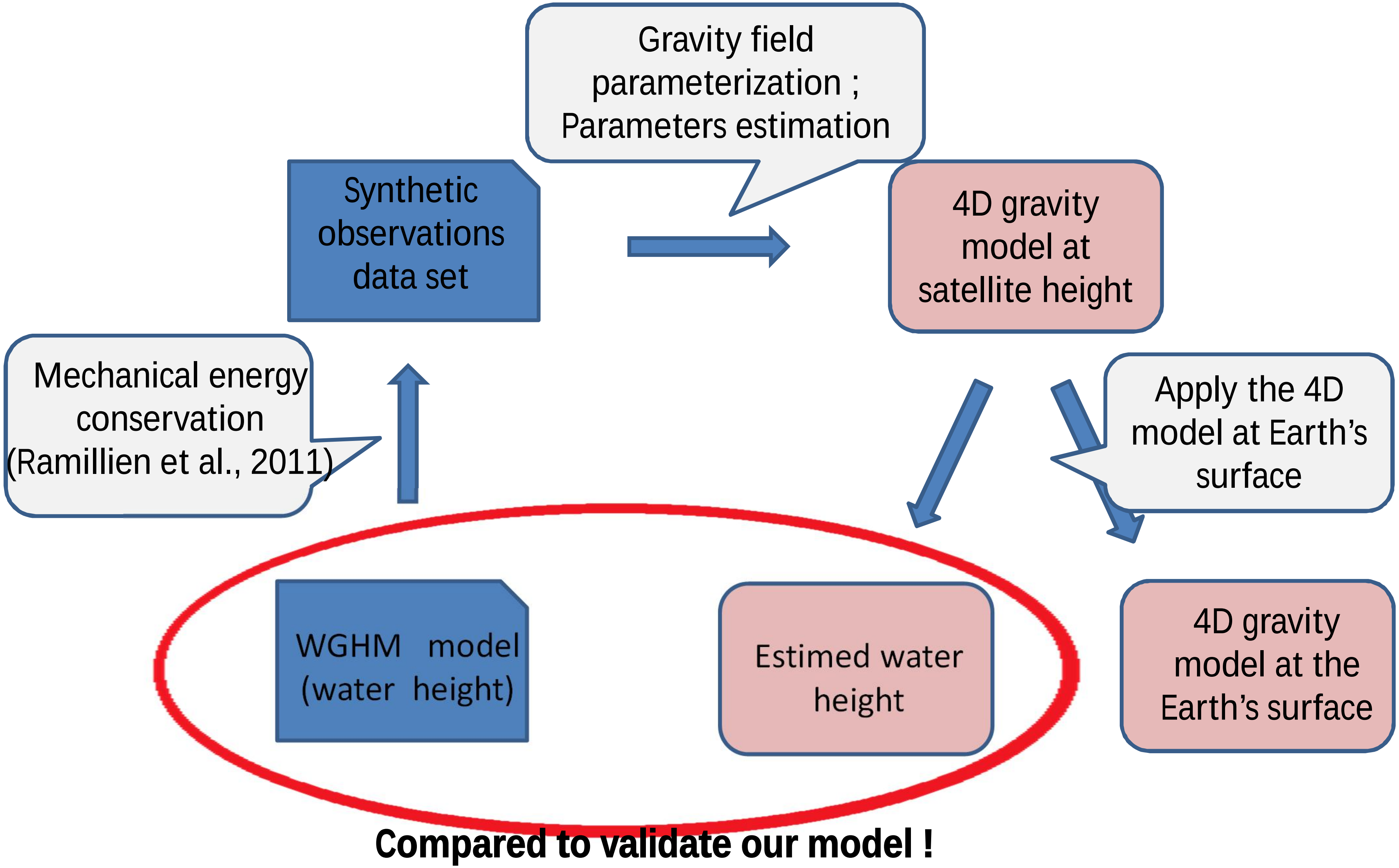
Schema of the work



Synthetic test parameters

- WGHM: year 2005, 0.5 ° resolution
- Synthetic data: 1 DDP / 5 seconds

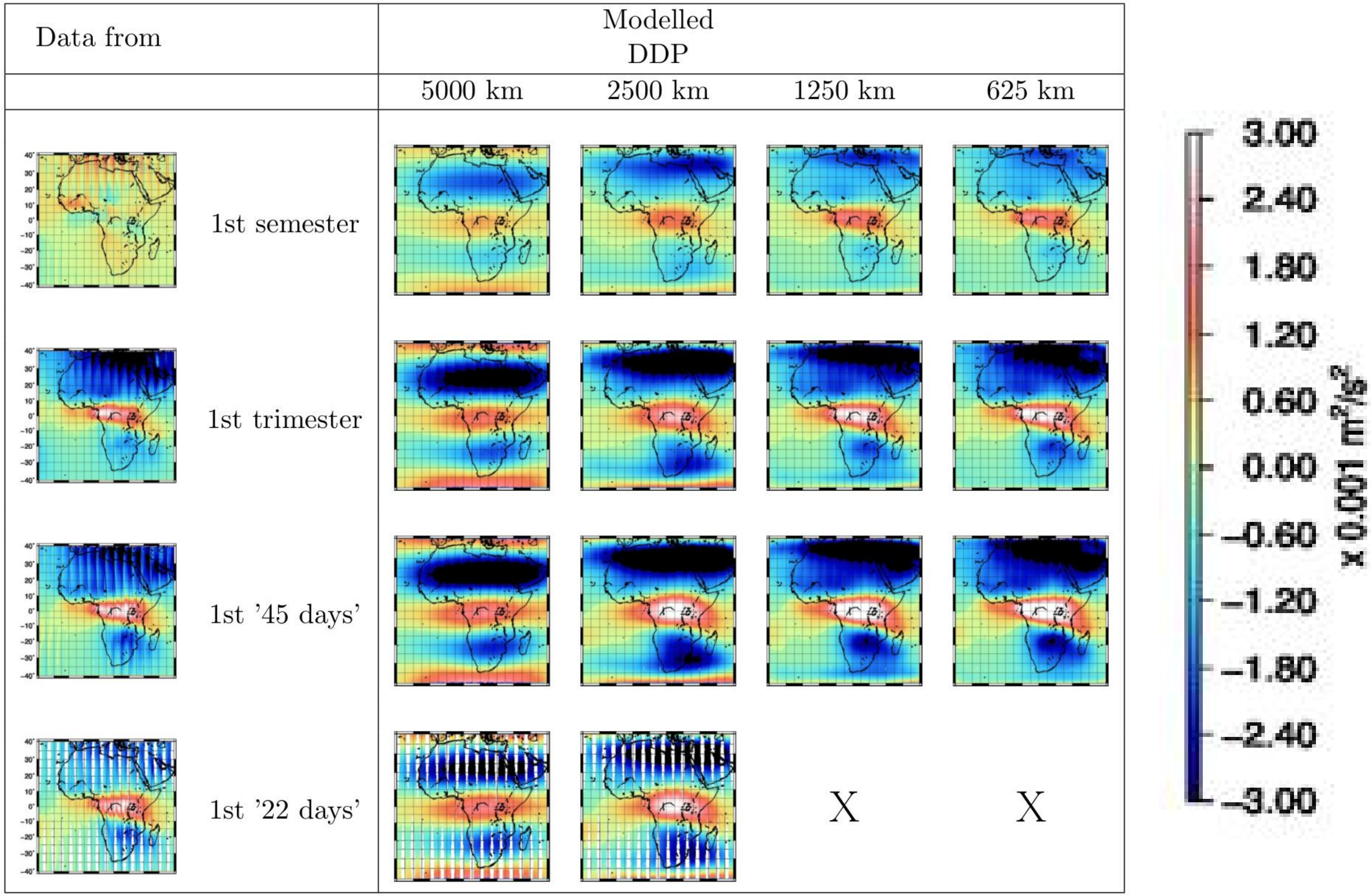
  → Two cases: noise-free, then white noise.

- 4D wavelets basis

	1 Y	6 m	3 m	1.5 m	22 d
5000km	YES	YES	YES	YES	YES
2500km	YES	YES	YES	YES	YES
1250km	YES	YES	YES	YES	NO
625km	YES	YES	YES	YES	NO

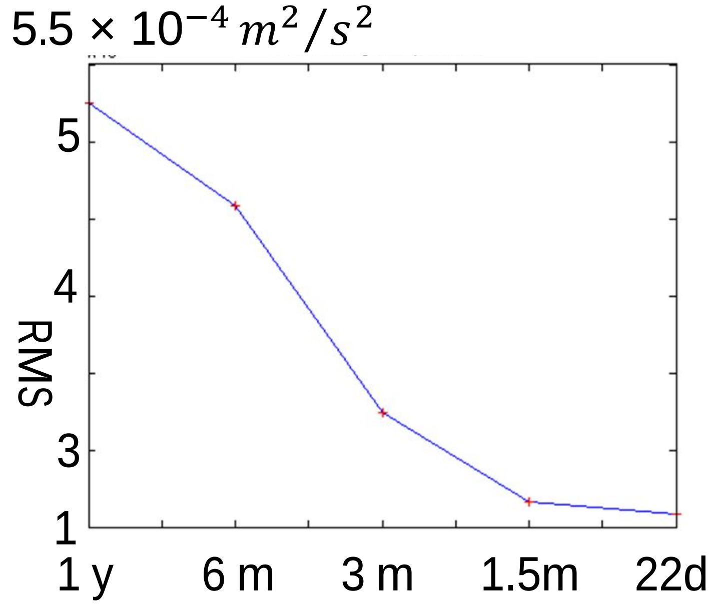
Each time period: 2250 / 133 spatial functions.

Estimated model at GRACE altitude

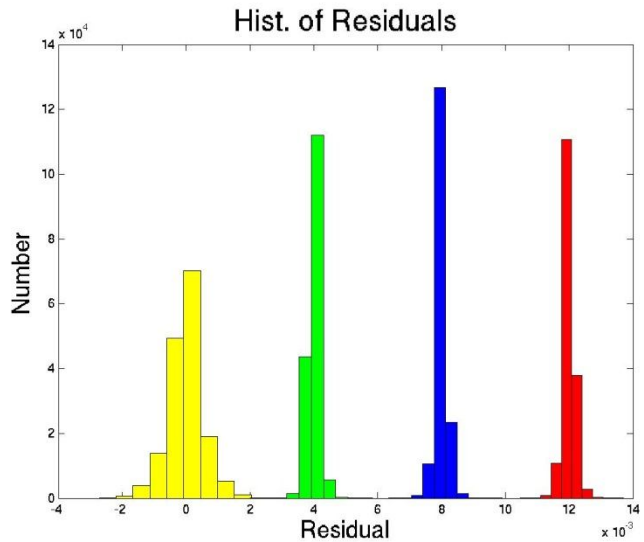


Data misfit

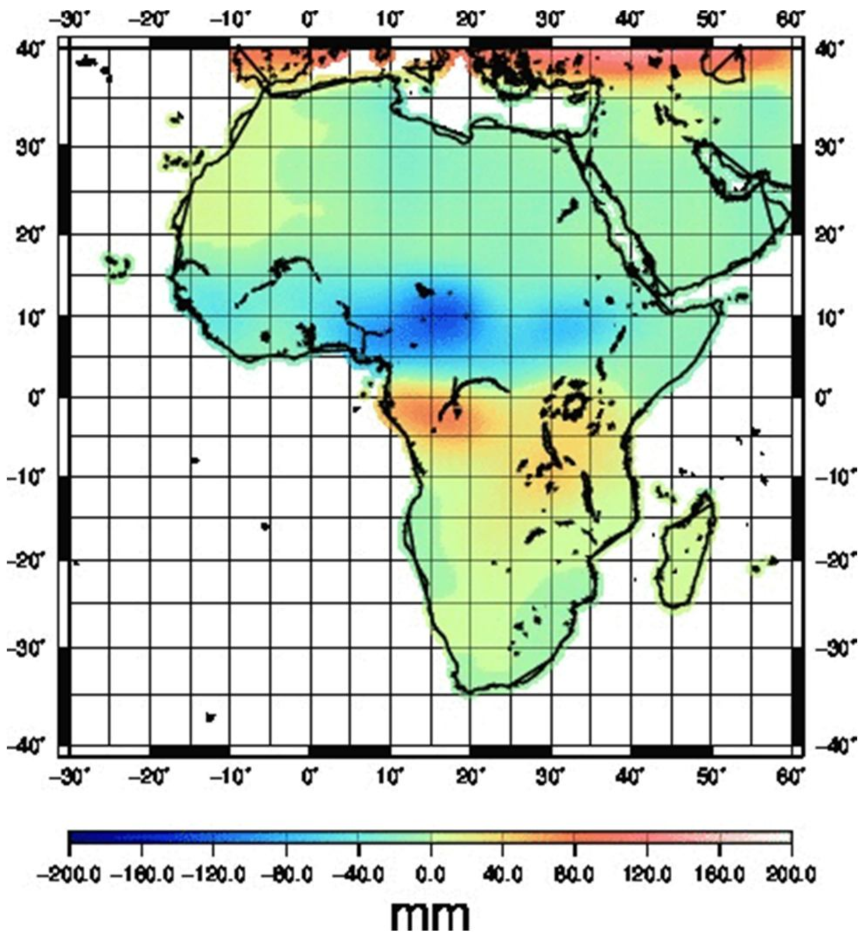
- The rms of residuals decrease when we increase the temporal resolution



- Distribution of the data residuals for each trimester of 2005.



The estimated water height

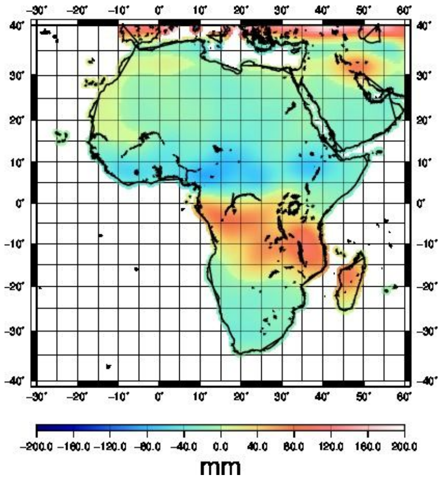


Distribution of the water height every 11 days, from 2005/01/01 to 2005/12/31.

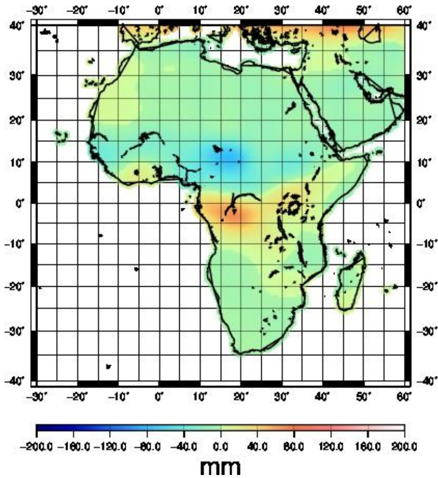
Comparison with WGHM

Estimated from the gravity model

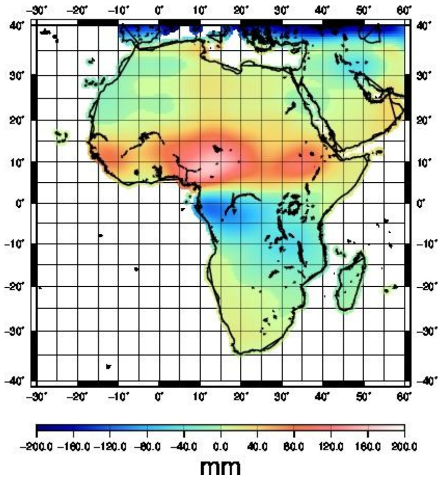
1st day to 11th day



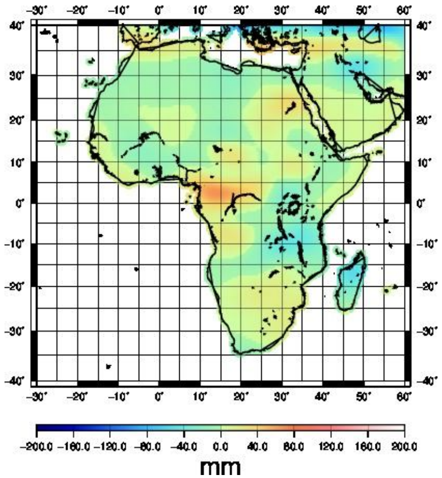
126th day to 132th day



239th day to 243th day

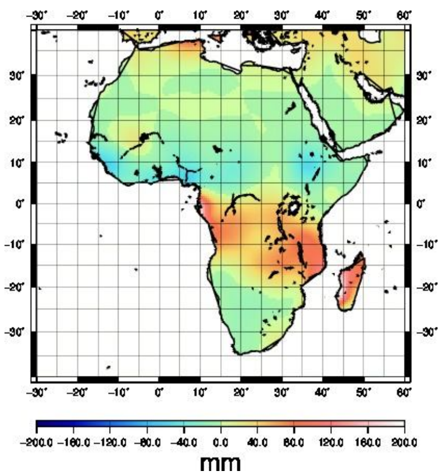


325th day to 336th day

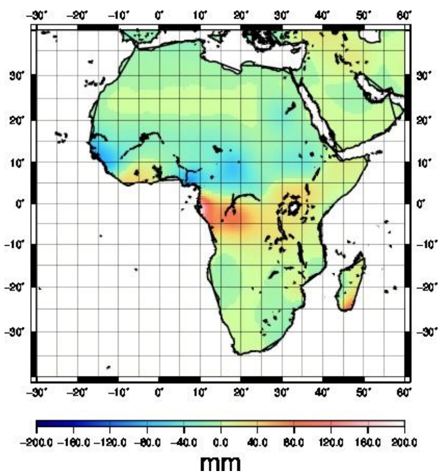


WGHM

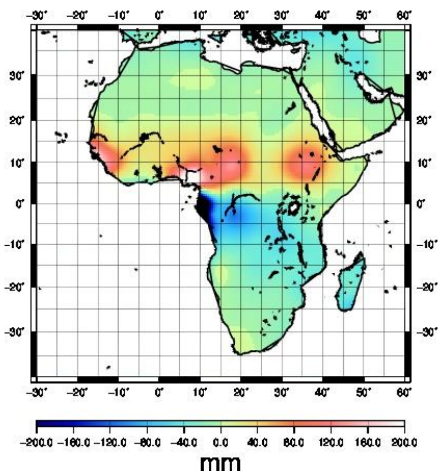
RMS: 3.4 cm



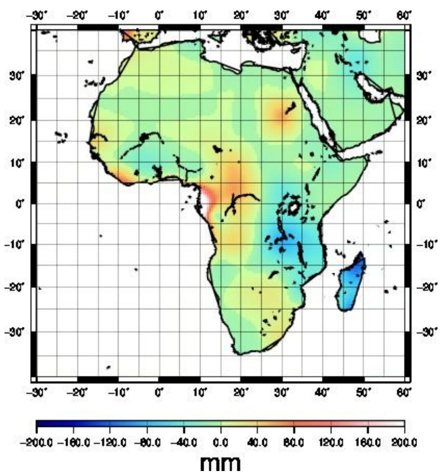
RMS: 3 cm



RMS: 3.8 cm

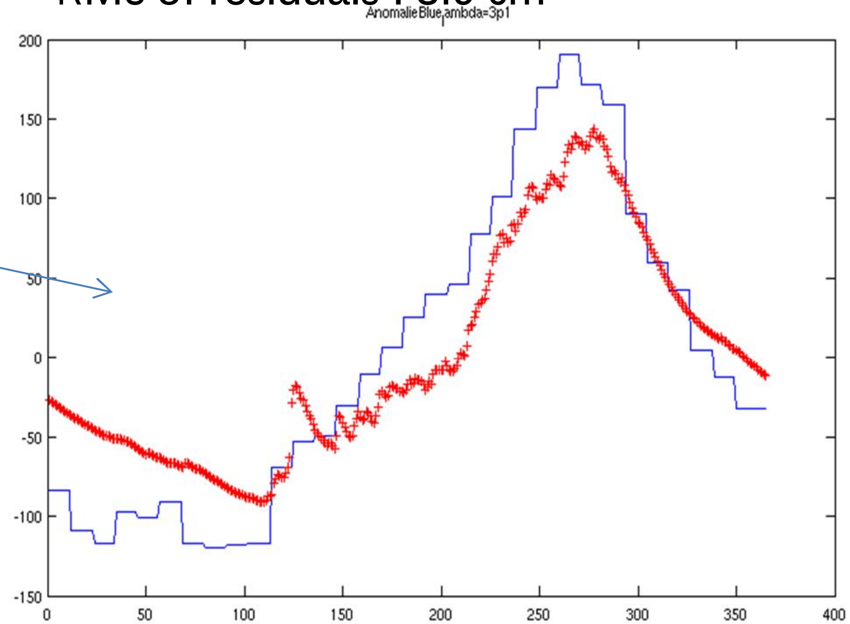


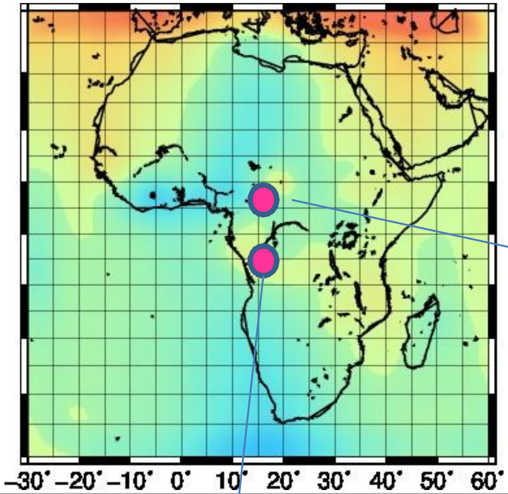
RMS: 3.3 cm

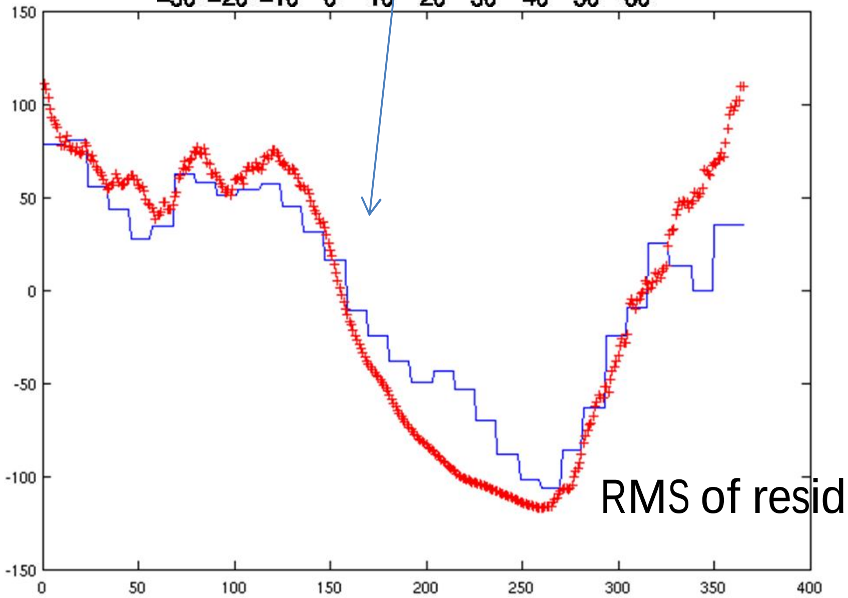


Time series at low latitude points

RMS of residuals : 3.9 cm





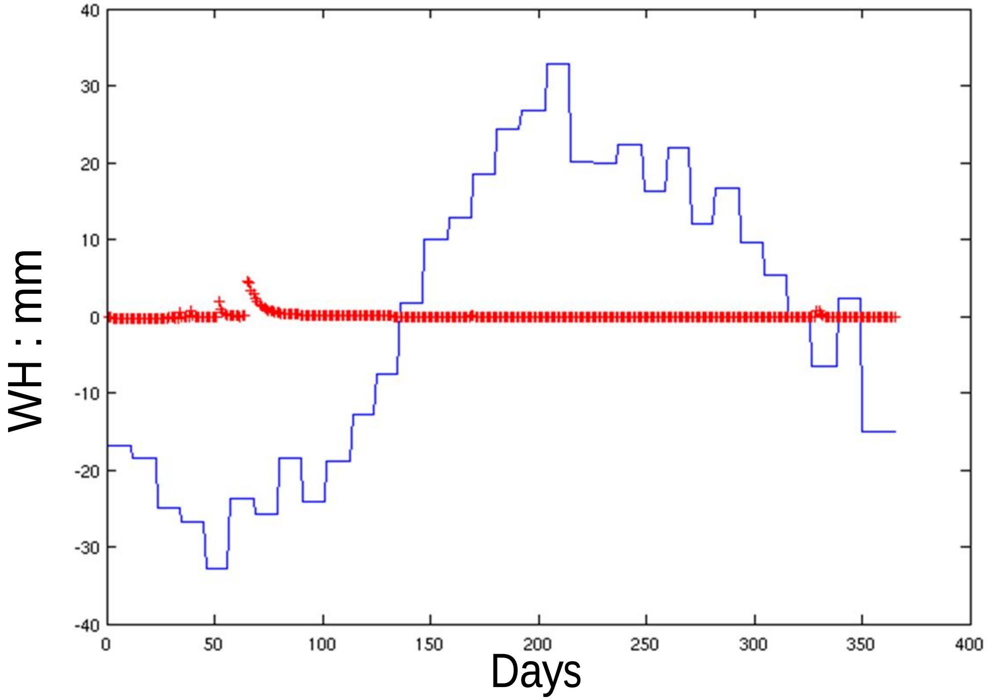


RMS of residuals : 2.5 cm

- Red one: WGHM model
- Blue one: estimated model

Improvements needed in desertic areas

- The model predicts a 3 cm water height seasonal signal.
- The regularization needs to be tailored regionally.



Conclusion

- We have developed a 4D approach to model Earth's gravity field in space and time
- We are able to recover water height variations over Africa with a ~3 cm accuracy: need to adapt the regularization regionally
- This method can later be applied to model real GRACE / GRACE Follow-On data

Thank you for your attention