Bayesian inversion of Glacial Isostatic Adjustment beyond linear viscoelasticity using Burgers rheology

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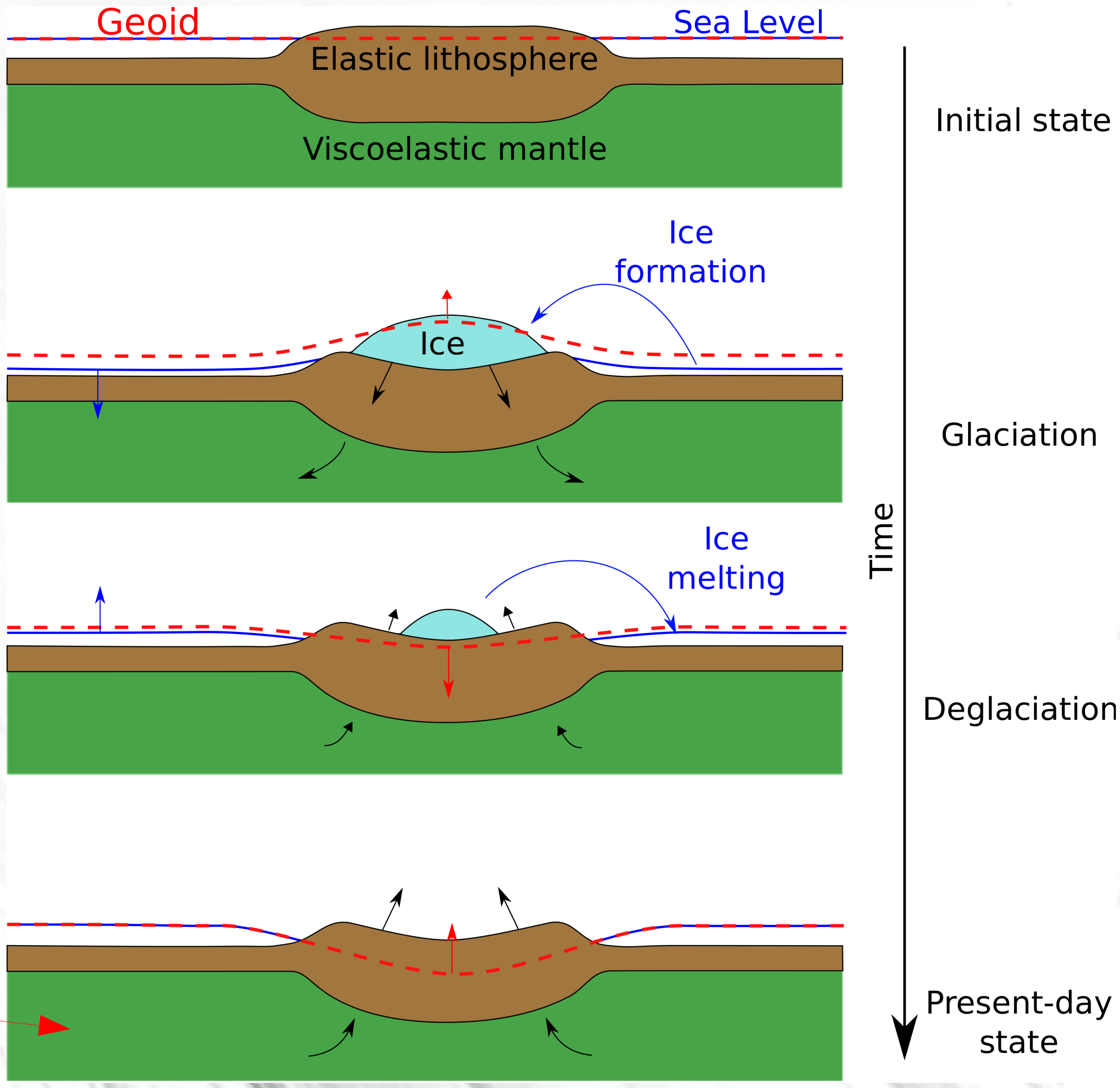
Introduction

GIA = signals generated by ice loading and unloading during glacial periods

- Ground displacement
- Gravity / Geoid changes
- Sea level geological records

Last glacial cycle:
120 kyr BP to present-day

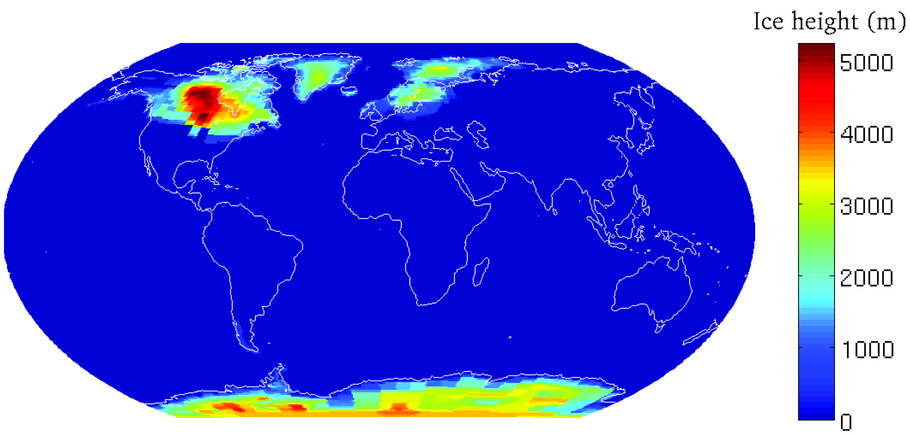
Choice of mantle rheology: Determining, yet commonly only Maxwell model is used



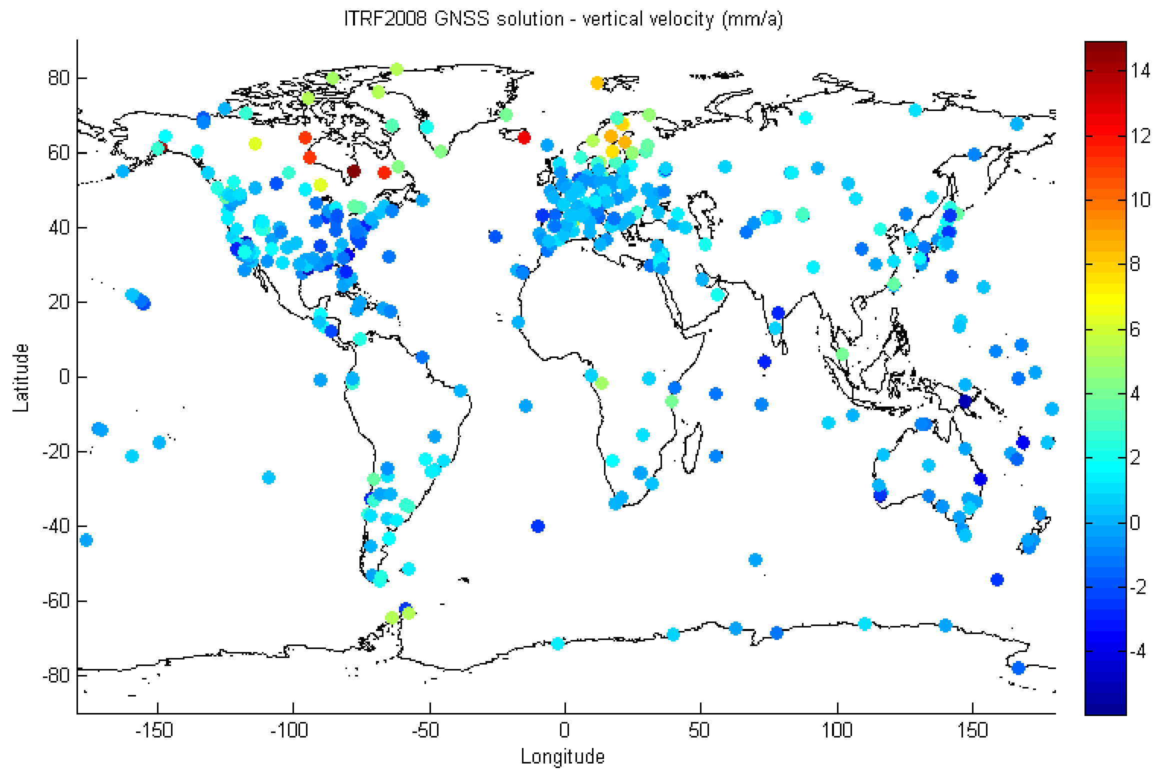
Some orders of magnitude

- Timescale : ~500 yrs – 120,000 yrs

Ice-5G model



Peltier (2004)

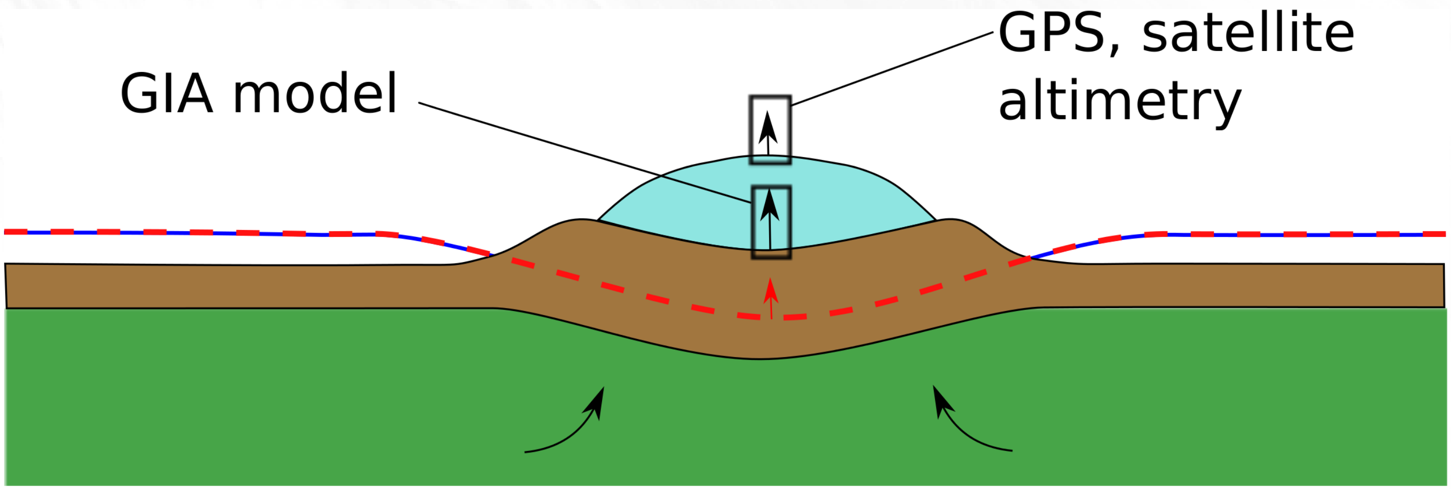


Paulson et al. (2007)

- Ice thickness : up to 3-5 km at Last Glacial Maximum (~20,000 yrs ago)
- Present-day vertical displacement : up to 10 – 20 mm/yr

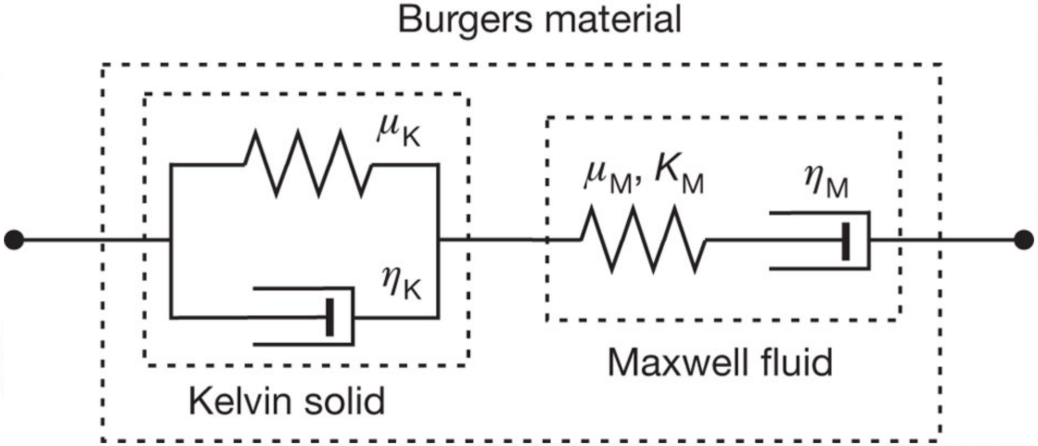
Interest of GIA modeling

- Mantle rheology (invert the viscosity profile)
- Correct gravity measurements (GRACE, GOCE)
  => study present-day sea level
- Correct GPS measurements
- Estimate present-day ice melting

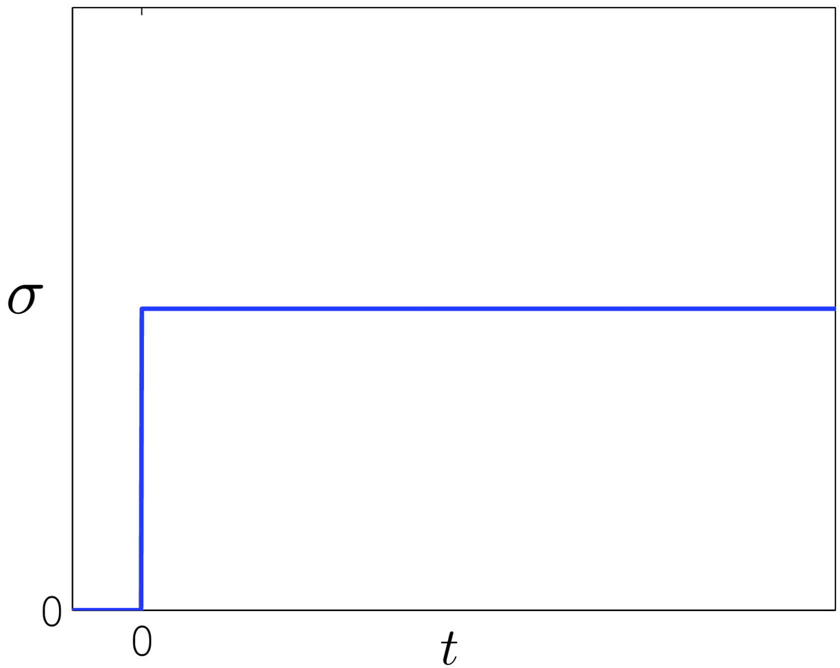


- Learn about the evolution of ice caps during the last glacial cycle

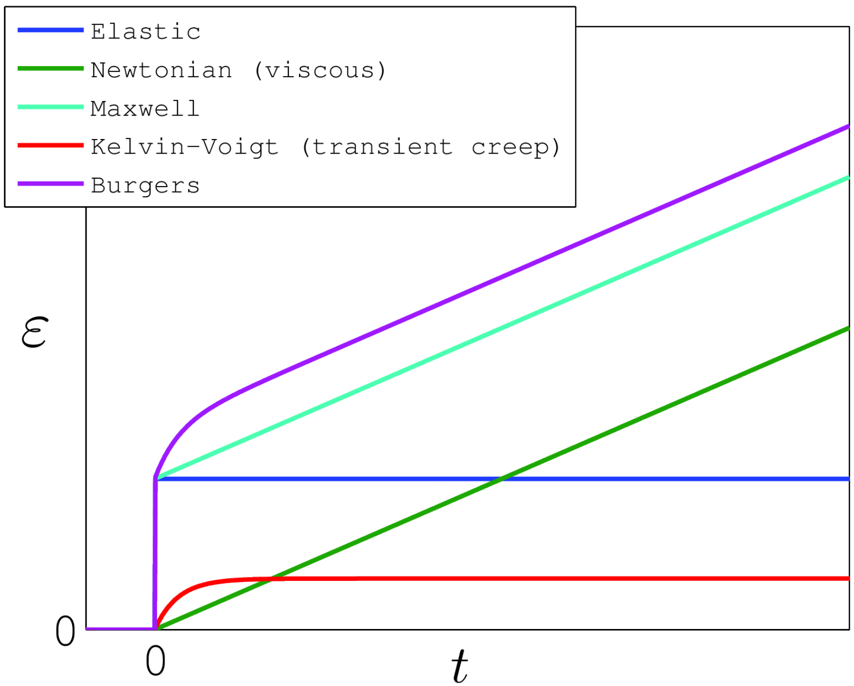
Burgers rheology



K. Wang et al. (2012)



Imposed **stress** (or force)

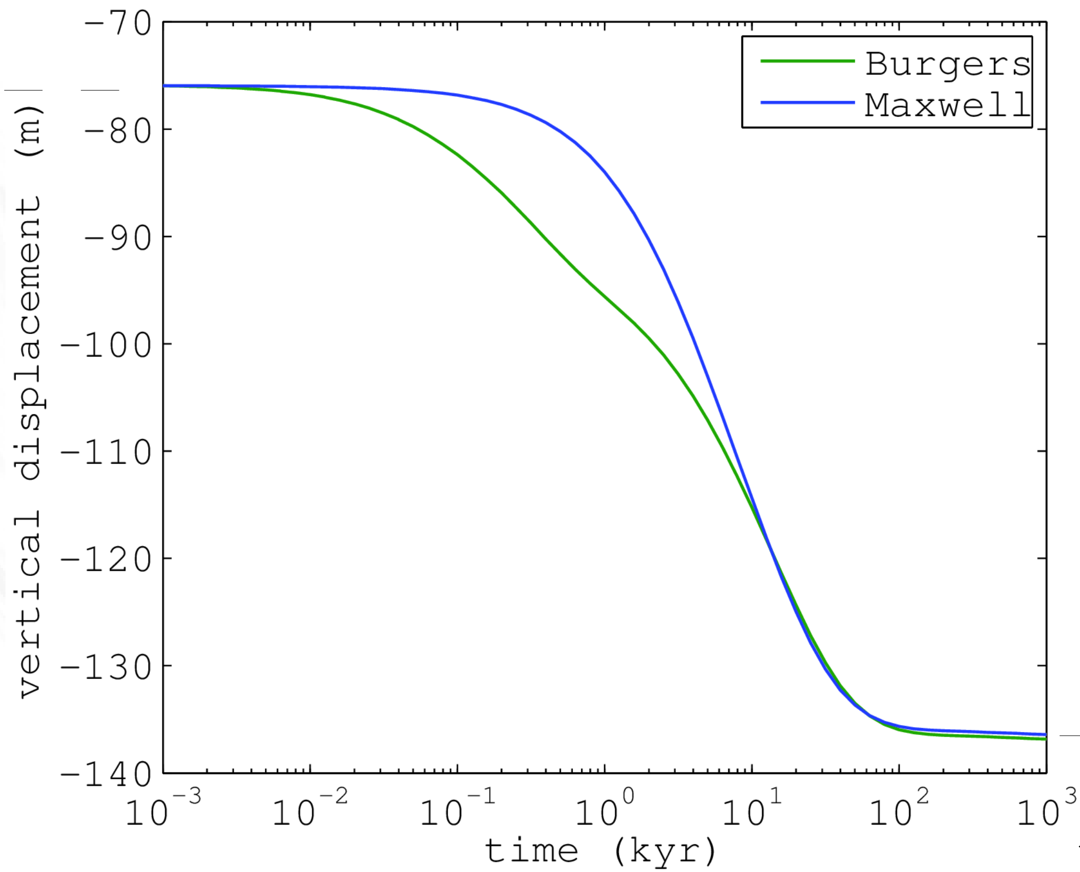


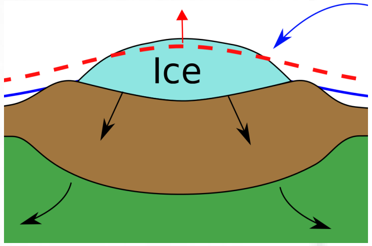
Deformation response

Rheology vs Isostasy

Response to the addition of mass at t=0

Elastic response





Isostatic equilibrium

← Log scale !

Burgers model accommodates deformation faster
In order to fit the same observation => higher viscosity or more ice

Why Burgers rheology ?

- Minerals physics experiments (Ivins & Sammis 1996) :
  1 mineral = Maxwell material
  Mix of **2 different Maxwell materials** = **Burgers material**

  viscosity = f(P, T°, Water content, chemical composition, grain size)
  These parameters are heterogeneous in the mantle
  => The mantle rheology should be at least Burgers !

- Post-seismic deformations :
  observation of a Burgers response of shallow mantle material
  (Pollitz 2005, Trubienko et al. 2013, K. Wang et al. 2012)

The main question

The traditional choice in Glacial Isostatic Adjustment models is to take Maxwell rheology.

What happens if we input Burgers rheology into GIA models ?

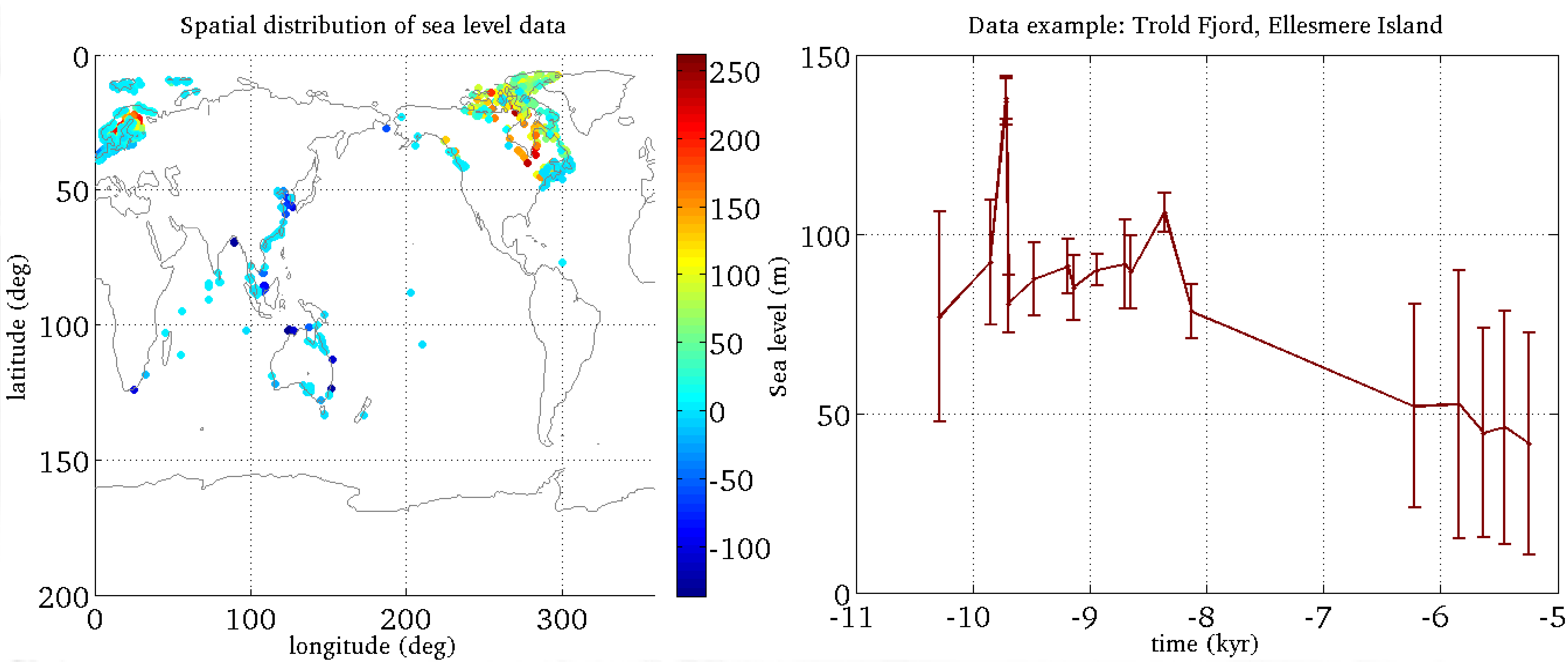
Inversion

- Forward model = knowing parameters, predict data
- Inverse model = knowing data, predict parameters
  Goal: get a range of possible values, a best-fitting model, and estimate trade-off effects

Dataset inverted :

- Sea level records (corals, raised beaches, fossils, …)
- Gradiometry (derivatives of gravity field obtained by satellite gravimetry)

Sea level data

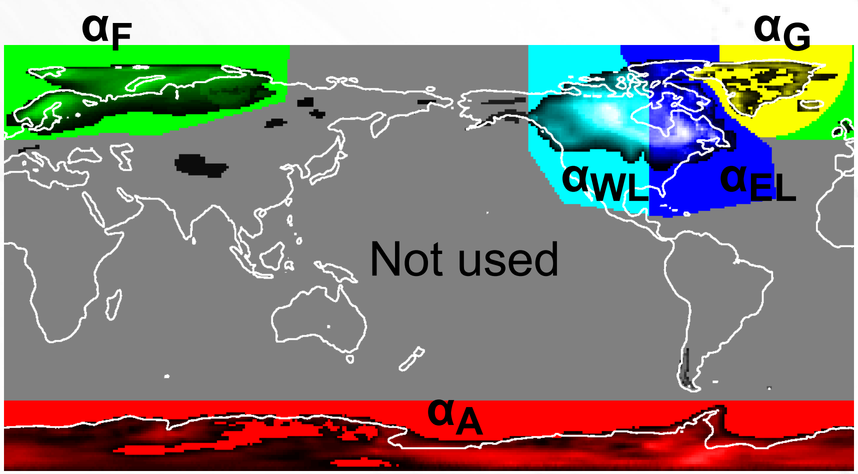


Inversion

Probabilistic approach (= Bayesian) :

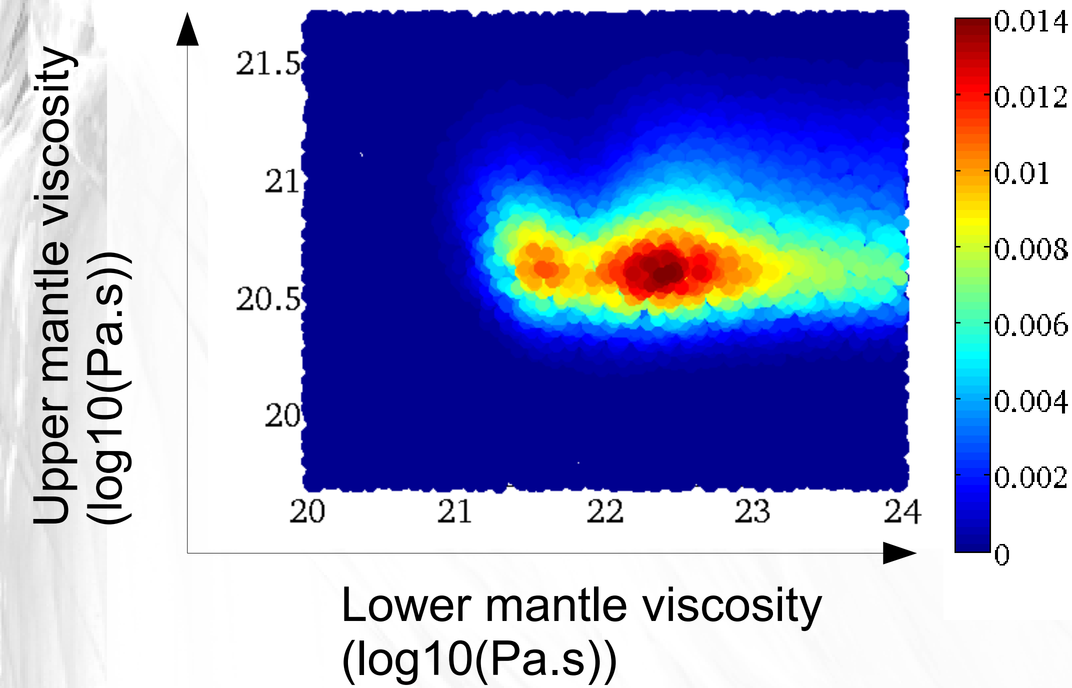
$$p(\boxed{x_1, x_2, ..., x_N}) = exp\left(-\frac{1}{2\sigma^2_{data}}\sum_{i=1}^{N_{data}} w_i^2 \left(\frac{data_i - \boxed{model_i}}{\epsilon_i}\right)^2\right)$$

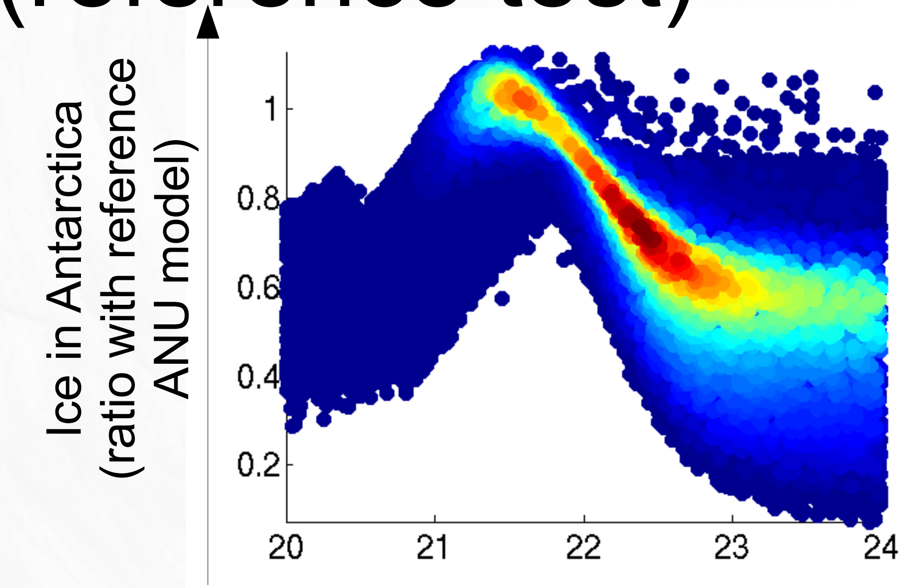
- η_{UM}: long-term viscosity of the upper mantle (Pa.s)
- η_{LM}: long-term viscosity of the lower mantle (Pa.s)
- T_e: elastic thickness of the lithosphere (km)
- η_1/η_2: long-term to short-term viscosity ratio for each layer of the mantle in Burgers models
- μ_1/μ_2: additional deformation quantity ratio for each layer of the mantle in Burgers models
- α_X: multiplicator for the amount of **ice** in region X

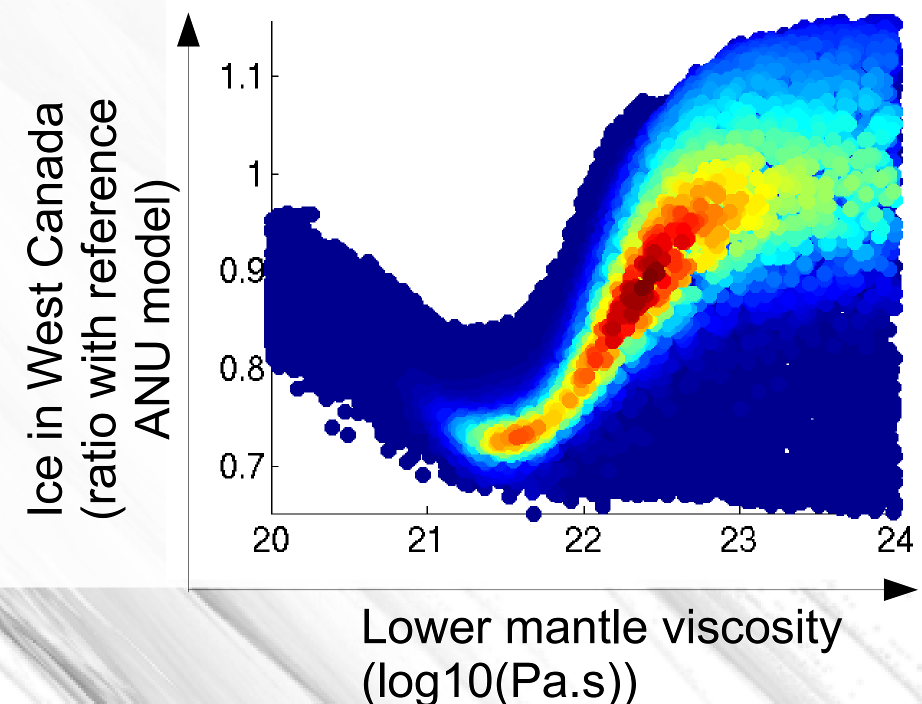


Map of the different ice regions adjusted during inversion. Reference is the ANU model (Lambeck et al., 2010)

Maxwell inversion (reference test)

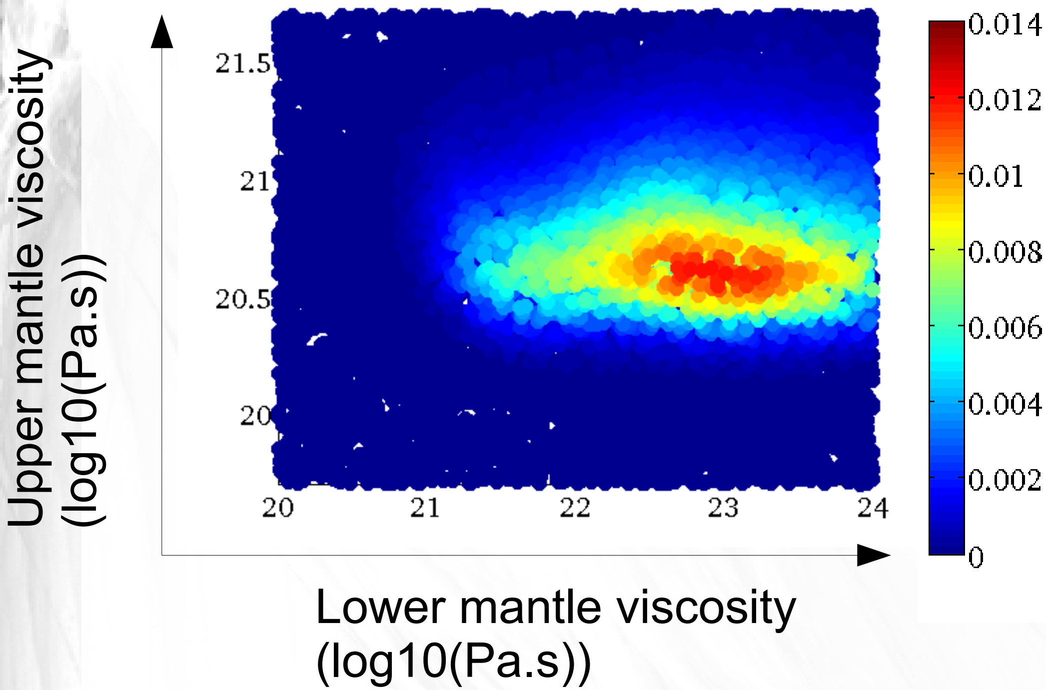


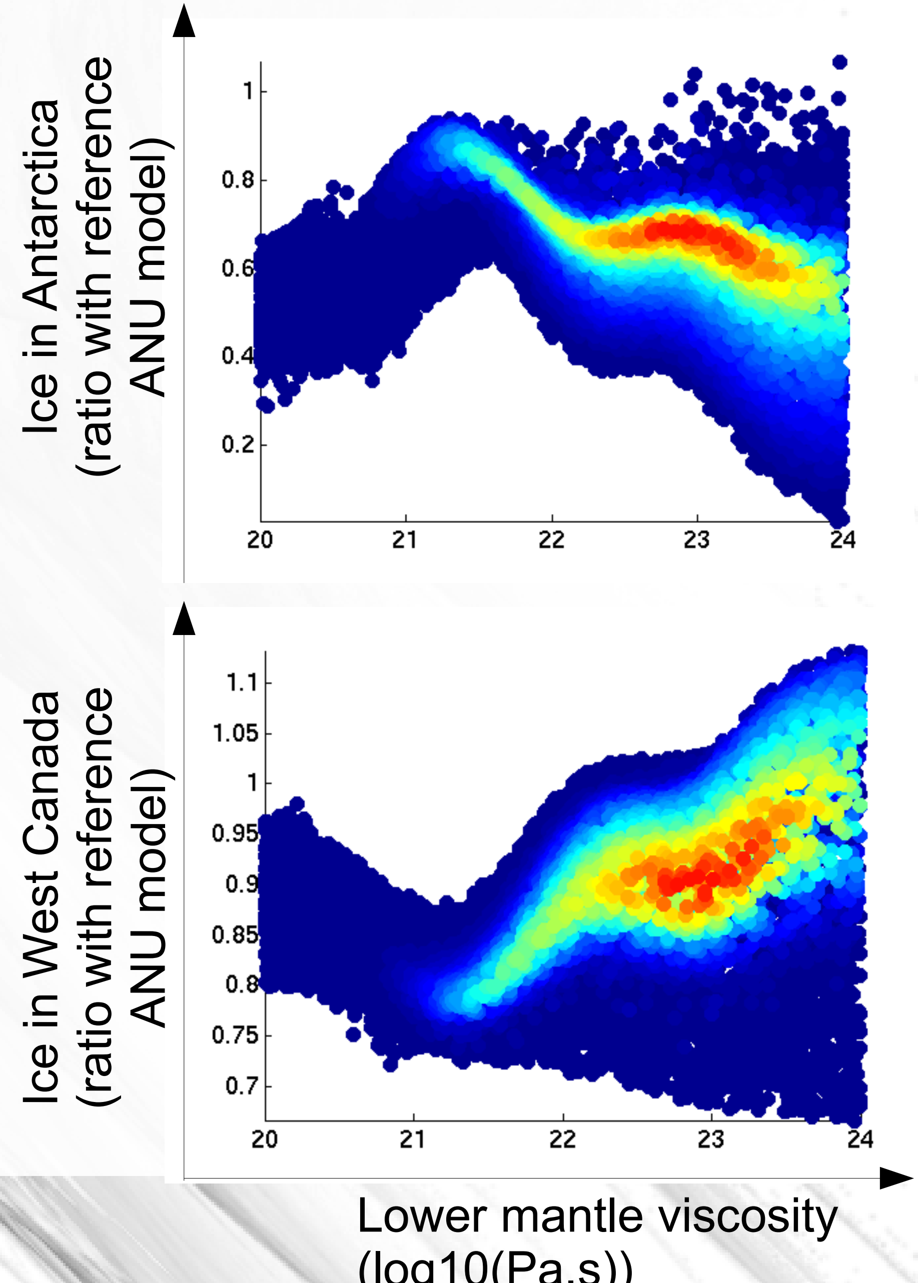




- Upper mantle viscosity = $5 \pm 2 \times 10^{20}$ Pa.s
  Uncorrelated to other parameters
- Lower mantle viscosity = **Two solutions** :
  $3\ 10^{21}$ and $3\ 10^{22}$ Pa.s
  Highly correlated with ice coefficients

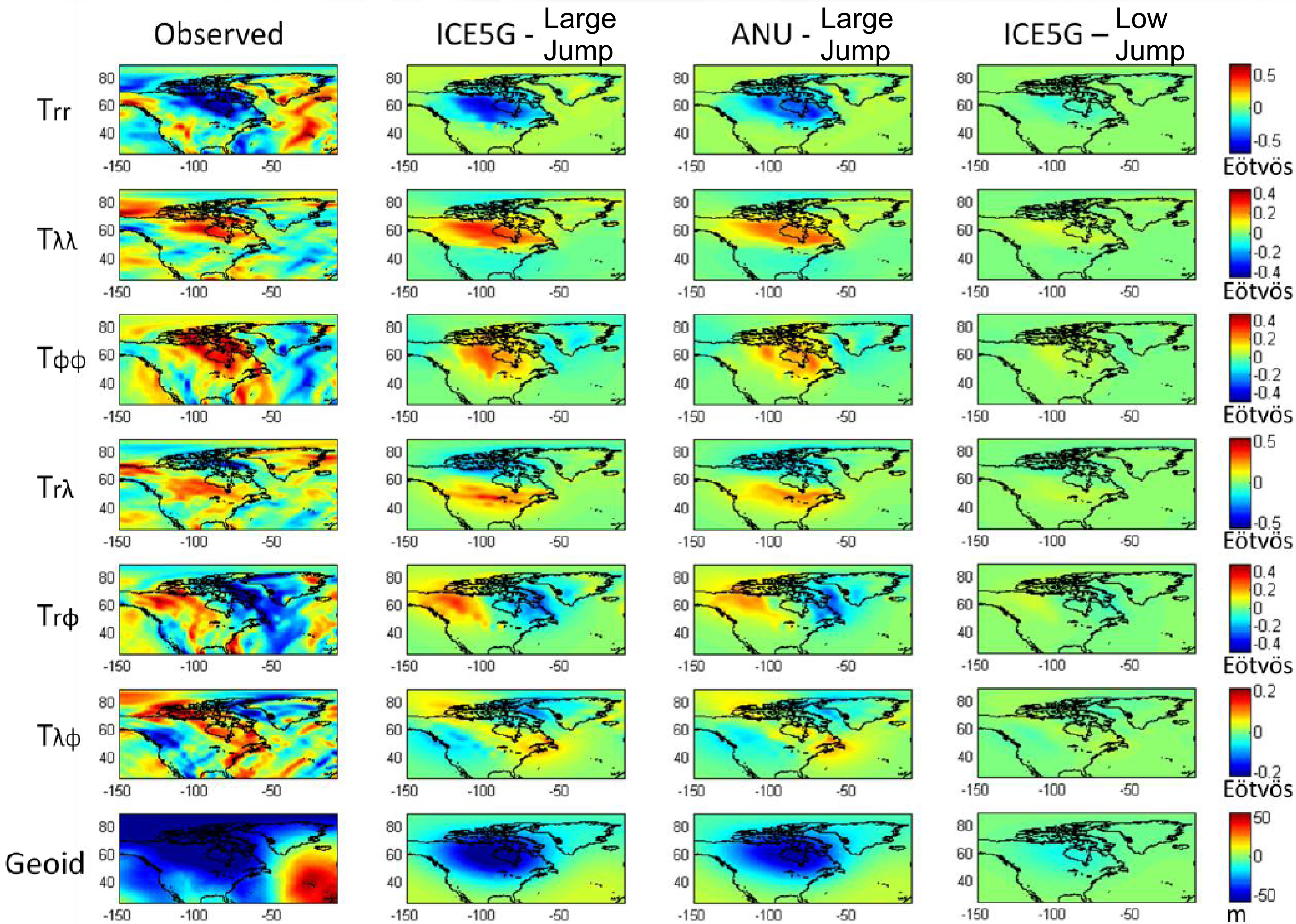
Burgers inversion (η_1/η_2 and $\mu_1/\mu_2 = 5$)



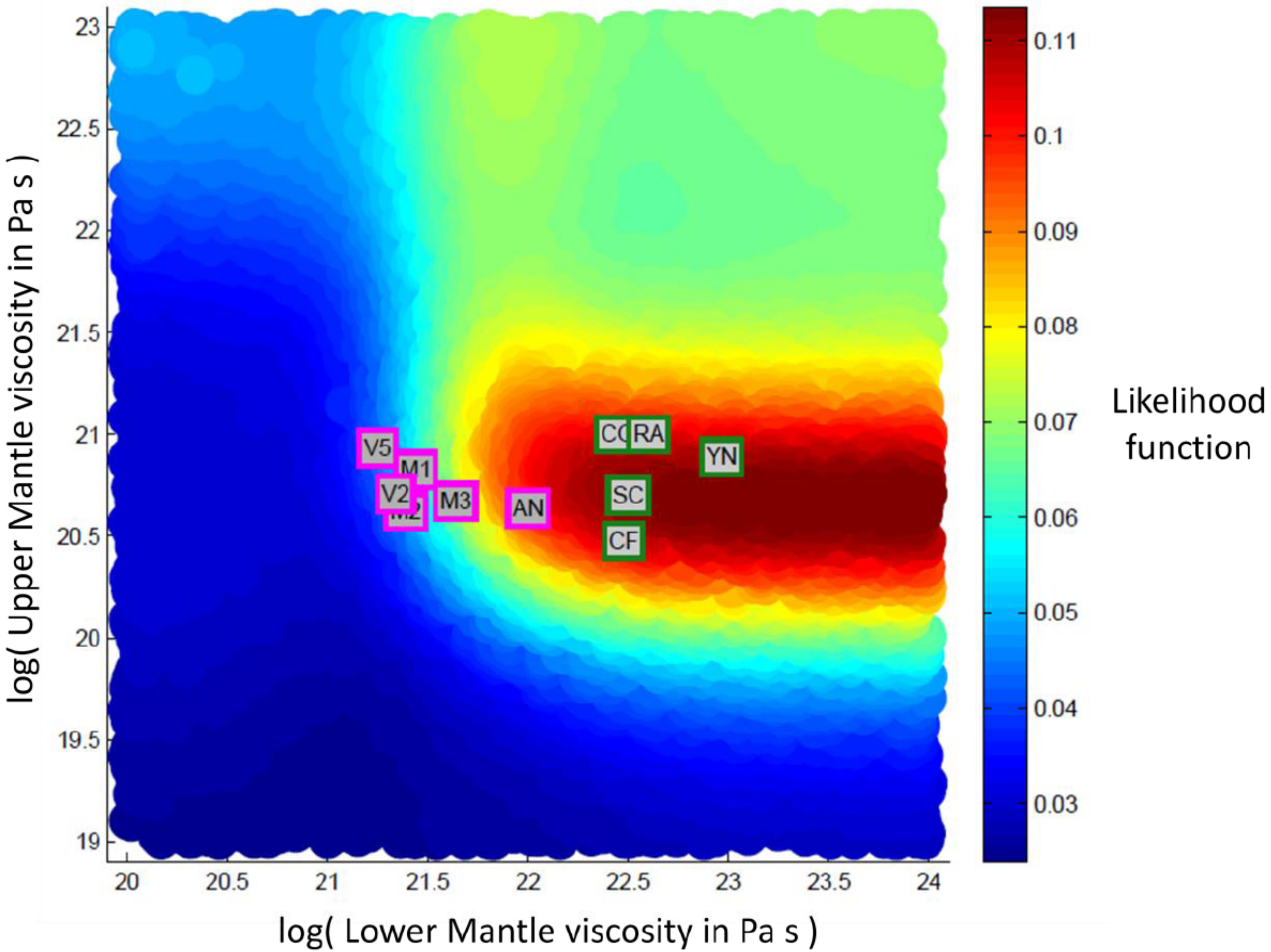


Only solutions with high lower mantle viscosity remain => ice scenario with less ice in Antarctica, more in Canada and Europe

Gradiometry forward modeling



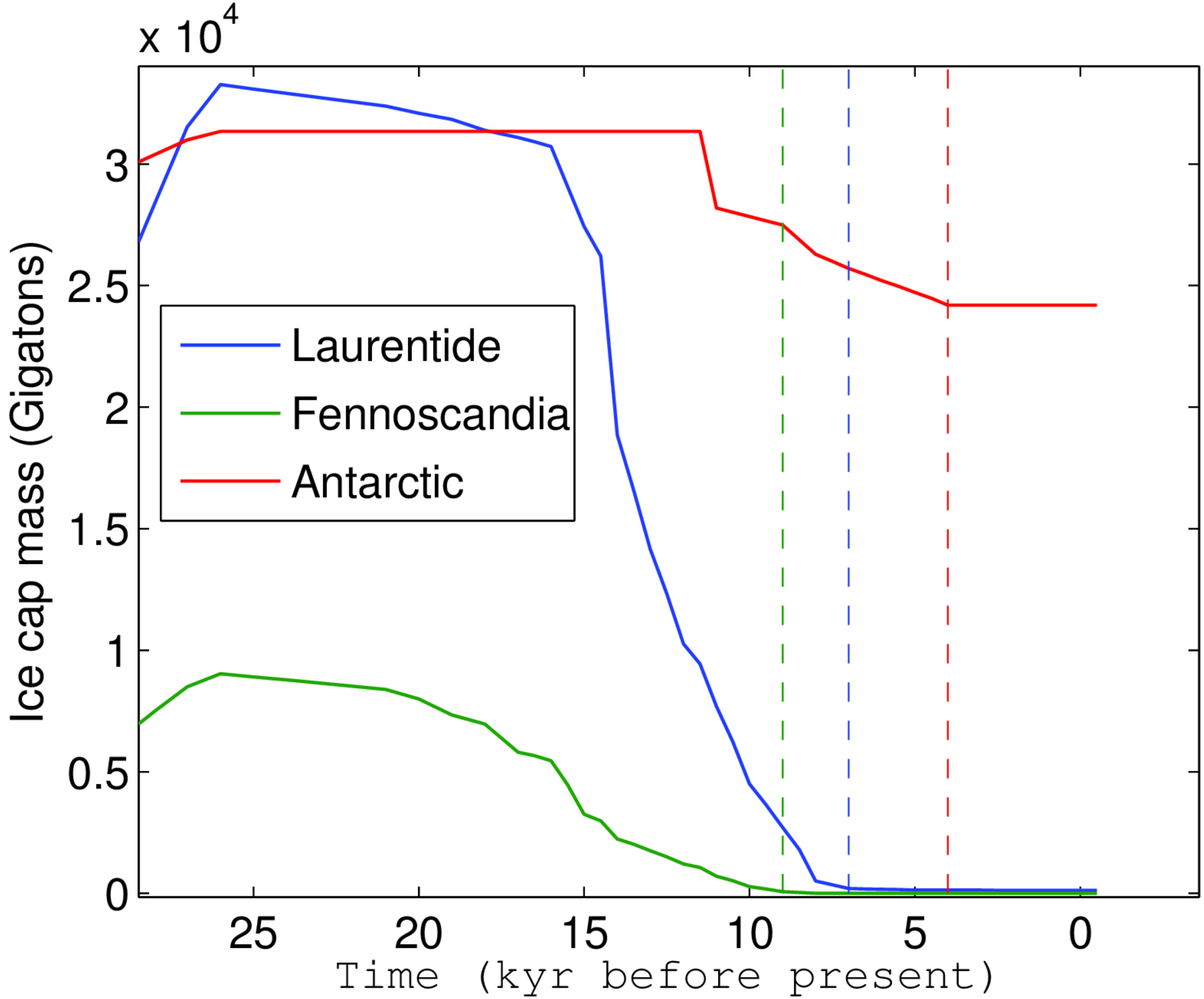
Gradiometry inversion

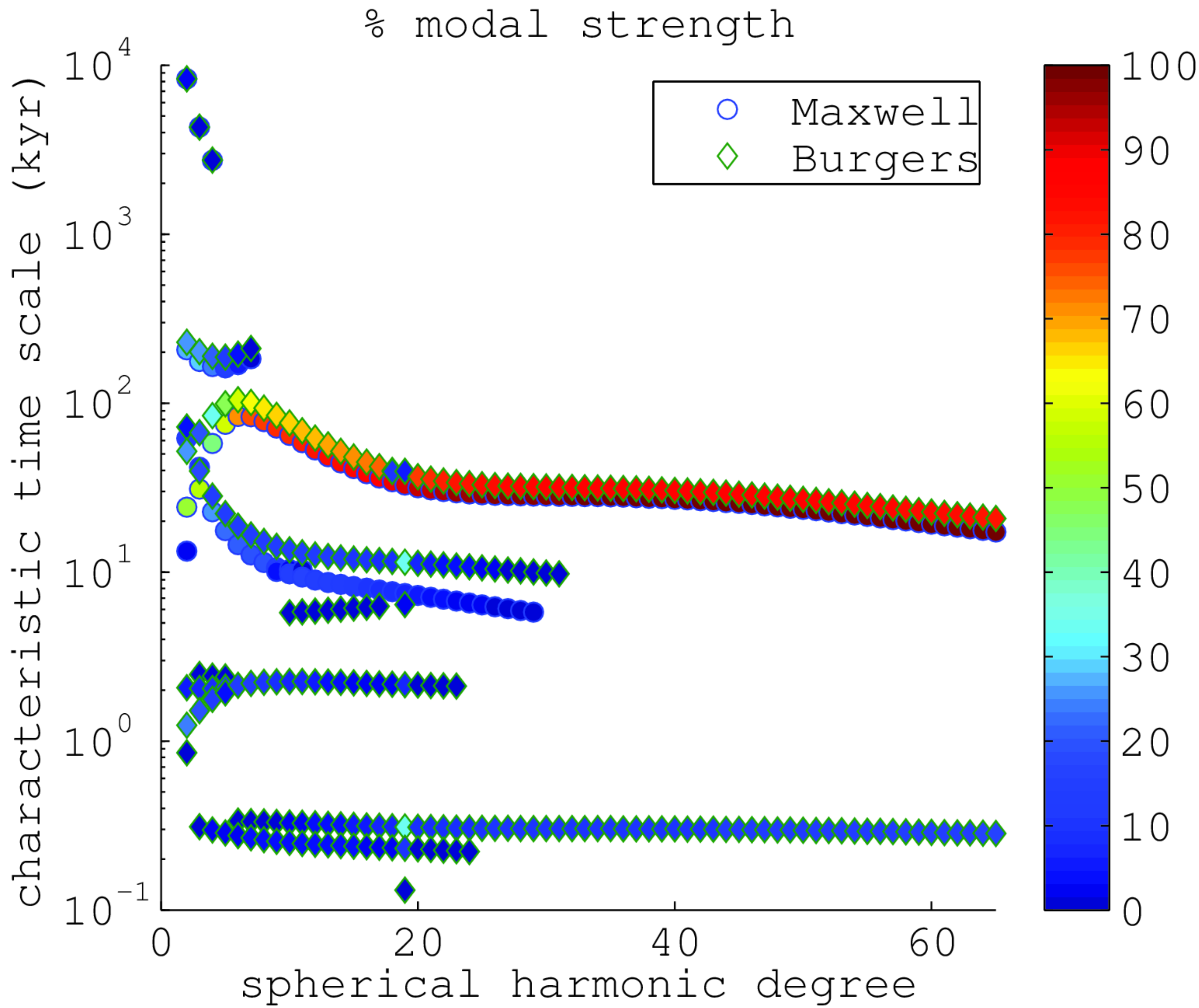


Conclusions & Perspective

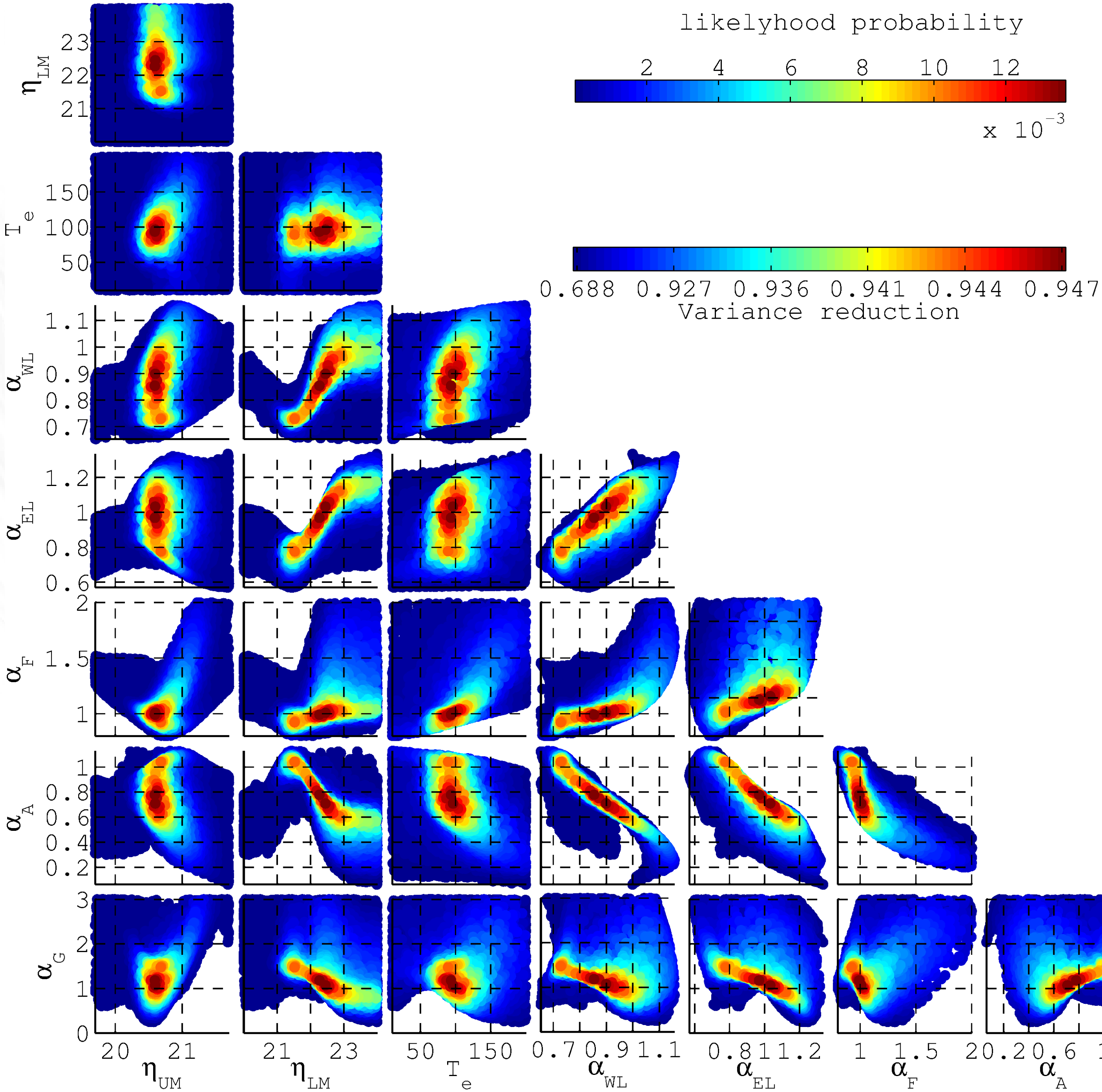
- Burgers rheology and Maxwell provide similar fit to sea level data
- Upper mantle viscosity constrained to $5 \pm 2 \times 10^{20}$ Pa.s and lithospheric thickness to 95 ± 10 km
- The **lower mantle** viscosity is highly **correlated** with **ice** caps distribution
- **High** long-term **lower mantle viscosity** is preferred using Burgers rheology
- **High lower mantle viscosity** also works better to **fit gradiometry** in Laurentide region
- Large interest for other geodetic data to **improve GIA inversion**, particularly **with horizontal GPS**
- Experiment with more parameters: asthenosphere, transition zone, more ice cap regions, ...

Thank you for your attention !

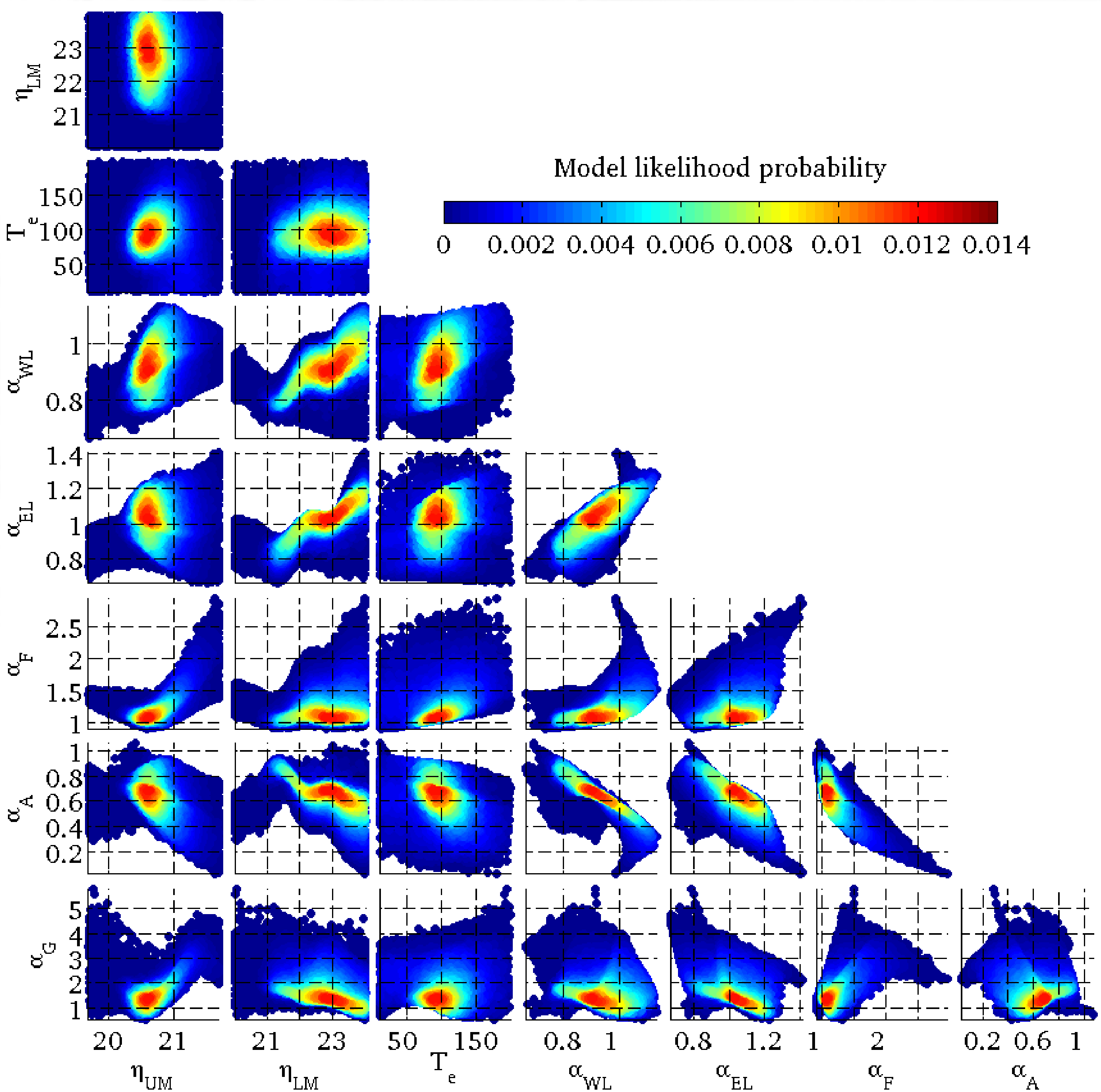

x 10^4
3
2.5
2
1.5
1
0.5
0
Ice cap mass (Gigatons)
Laurentide
Fennoscandia
Antarctic
25
20
15
10
5
0
Time (kyr before present)



% modal strength
characteristic time scale (kyr)
spherical harmonic degree
Maxwell
Burgers
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
0
20
40
60
0
10
20
30
40
50
60
70
80
90
100



likelyhood probability
2 4 6 8 10 12
x 10^-3
0.688 0.927 0.936 0.941 0.944 0.947
Variance reduction
η_LM
T_e
α_WL
α_EL
α_F
α_A
α_G
η_UM



Model likelihood probability
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
η_{LM}
T_e
α_{WL}
α_{EL}
α_F
α_A
α_G
η_{UM}
η_{LM}
T_e
α_{WL}
α_{EL}
α_F
α_A